VSB – TECHNICAL UNIVERSITY OF OSTRAVA
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Determination of Credit Risk for Debt Assets Portfolio

Determinace kreditního rizika u portfolia dluhových aktiv

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1. Introduction
2. Description of the Financial Risk
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4. Determination of Credit Risk by Selected Models
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List of Abbreviations
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References:

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Contents:

1 Introduction

Risk management is a key area from the perspective of banks to assure uncertainty will not have a bad influence on banks' performances because risks are essential in the process of financial transaction and the quality of risk management has a significant impact on the overall stability and profits of banks. There are several categories of risks in the banking sector, but the most important risk is credit risk.

The main objective of this thesis is to determine and then compare the value of regulatory capital requirement for unexpected losses from credit risk of ten debt assets portfolio under Basel agreements, including Basel I, Basel II, and Basel III, and the value of economic capital by using the CreditMetrics™ model.

The thesis can be divided into the theoretical and the practical part. The theoretical part describes basic financial risks firstly and credit risk is emphasized. Then several complex models, especially the CreditMetrics™ model, for credit risk management are introduced. The process of determining the value of economic capital by using the CreditMetrics™ model is analyzed in a whole subchapter. The last will focus on the description of the Basel agreements on capital adequacy.

In the practical part, credit risk is calculated with a portfolio consisting of ten selected bonds traded on the Frankfurt Stock Exchange (FSE). The nominal value of the overall portfolio is 10 million euro and the time horizon for determining credit risk is one year. The value of regulatory capital requirement to cover unexpected losses will be subsequently determined under the Basel agreements, both the standard approach and the foundation internal ratings-based approach included. Then follows the calculation of economic capital of the portfolio by the CreditMetrics™ model. In conclusion, the results obtained by different methods will be compared and interpreted in detail.

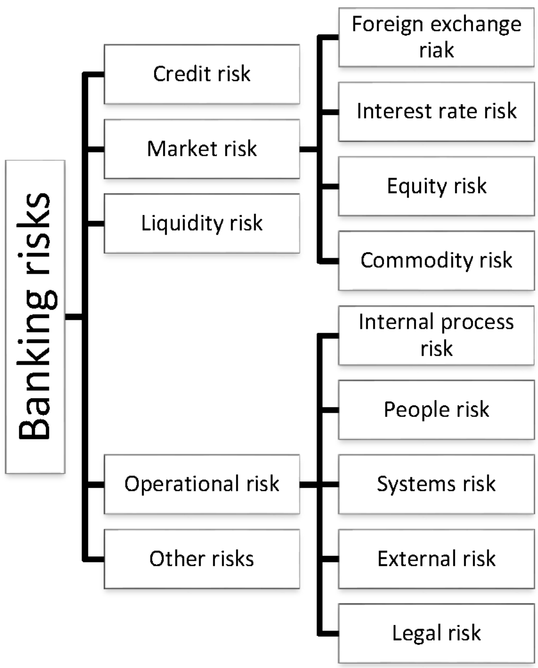
2 Description of the financial risk

In this chapter, the four main types of banking risks are described respectively, including credit risk, market risk, liquidity risk, operational risk, and other risk types. Among these banking risks, the credit risk is the oldest and largest risk, and we are supposed to pay more attention to it.

Banks are widely considered to be stable and reliant. However, when banks accept deposits from one group and use funds to provide credit products to another group, risks are created. Banks are faced with different types of risks all the time that usually have a potentially negative effect on their operations. Thus, in order to minimize the banking risks and make the banks well-operational, the risk management in banking emerges at a historic moment, including measure, manage, and monitor the banking risks.

There are four typical types of banking risks: credit risk, market risk, liquidity risk, and operational risk.

Figure 2.1: Banking risks



2.1 Credit risk

Credit risk is the potential loss for a bank when a bank borrower will fail to meet its obligations, such as paying interest on the loan and repaying the amount borrowed, in accordance with agreed terms. It is the largest risk that most banks are faced with.

For example, in China Construction Bank (CCB), the risk-weighted assets and capital requirements are presented in *Tab 2.1*. Note that the credit risk-weighted assets were calculated with the regulatory weight approach, the market risk-weighted assets were calculated with the standardized approach, and the operational risk-weighted assets were calculated with the basic indicator approach.

Tab 2.1: The risk-weighted assets and capital requirements in CCB in 2014 (In millions of RMB, except percentages)

	Risk-weighted assets	Capital requirements	%
Credit risk-weighted assets	8,739,574	699,166	85.65%
Market risk-weighted assets	54,302	4,344	0.53%
Operational risk-weighted assets	915,727	73,258	8.97%
Additional risk-weighted assets	494,040	39,523	4.84%
Total	**10,203,643**	**816,291**	**100.00%**

Source: Annual report 2014 of CCB. 61p.

Modern banks commonly face credit risk in a number of other financial instruments such as interbank transactions, financial futures, options, bonds, equities, swaps, and so on. This type of credit risk is referred to as counterparty credit risk (CCR). It is usually used to indicate the credit risk raised by off-balance sheet products and guarantees.

2.1.1 Types of credit risk

Classifying credit risk does help to identify and understand it better. Although credit risk can be classified in different classification methods, the following classification is relatively widely accepted:

- default risk,
- credit spread risk,
- downgrade risk.

Default risk

Default risk is the risk that the debtor fails to meet his or her obligations of interest and principle. It may have an impact on all credit-sensitive transactions. In these cases, the investor can refer to a credit risk report, which is based on the debtor's credit history and finances. Besides, both the default rate and the recovery rate will do help. The default rate is the percentage of a population of bonds that are expected to default, while the recovery rate indicates how much the investor can expect to get back if a default occurs.

Credit spread risk

Credit spread refers to the spread between securities, which are without risk, and certain bonds, which may be very risky. For example, the difference between yields on treasuries and A-rated bonds. Generally speaking, a firm must offer a higher yields on the bonds they issue than the government's bonds because of their worse credit. In poor performing economies, spreads usually tend to widen.

Downgrade risk

Downgrade risk occurs when there is the possibility that the rating agencies, such as Moody's, S&P and Fitch, will lower their rating of a particular debtor once a loan is issued. If one of these rating agencies downgrades a firm's rating, it may be more difficult for the corporation to pay.

2.1.2 Factors affecting credit risk

To quantify the credit risk, there are several major variables to consider: the financial health of the borrower; the severity of the consequences of default for the borrower and the creditor; historical trends in default rates; the size of the credit extension; and many macroeconomic factors, such as economy environment, institutional issues and legislation, and physical environment. However, among all these possible variables, four of them are usually considered having a strong impact on credit risk, namely:

- probability of default,
- loss given default,
- exposure at default,
- time horizon.

Probability of default

Probability of default, also abbreviated as POD or PD, refers to the chances that the borrower fail to maintain the financial capability over a particular time horizon, usually one year. Generally, in order to compensate for default risk, the higher the default probability a lender estimates a borrower to have, the higher the interest rate the lender will charge the borrower. On the other hand, the borrower also can reduce default risk by pledging collateral against a loan or a debt.

Credit rating is usually considered an indicator of probability of default. A credit rating can be assigned to any borrowers, such as individuals, corporations, and sovereign government. There are two major categories of credit rating, namely internal rating assessment and external rating assessment. The rating criteria are similar for both internal systems and external systems, namely they include qualitative and quantitative factors.

Internal rating assessment is conducted by financial firms themselves, especially banks. In order to ensure the reliable and consistent performance of their rating systems, banks would like to conduct an internal rating assessment by their own so as to establish a credit rating governance framework.

External rating assessment is conducted by credit rating agencies. Credit rating agencies (CRAs) evaluate the creditworthiness of various borrowers. Rating agencies rate both borrowers and the debt issues.

There are two categories of long-term credit rating, including both investment grade ratings and non-investment grade ratings. *Tab 2.2* shows ratings scales used by Moody's, Standard & Poor's, and Fitch respectively, three of the major credit rating agencies.

Tab 2.2: Long-term rating matrix

Investment grade ratings		**Non-investment grade ratings**	
Moody's	*Standard & Poor's, Fitch*	*Moody's*	*Standard & Poor's, Fitch*
Aaa	AAA	Ba1	BB+
Aa1	AA+	Ba2	BB
Aa2	AA	Ba3	BB-
Aa3	AA-	B1	B+
A1	A+	B2	B
A2	A	B3	B-
A3	A-	Caa1	CCC+
Baa1	BBB+	Caa2	CCC
Baa2	BBB	Caa3	CCC-
Baa3	BBB-	Ca	CC
		C	C
		C	D

Triple A (AAA or Aaa) is the highest rating, and investment grade ratings are generally from AAA/Aaa to BBB/Baa. Non-investment grade ratings generally from Ba1/BB+ to C/D, and D means in default.

However, instead of showing each rating's credit as a sole grade, it is better to overview them in one table. An example is shown as *Tab 2.3*, which is Standard & Poor's one-year ratings transition matrix for 2014. These results are usually obtained from a sample of a great number of firms over many years.

Tab 2.3: 2014 One-year transition matrix (%)

Initial Rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	90.81	8.33	0.68	0.06	0.12	0.00	0.00	0.00
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0.00
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0.00	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC/C	0.02	0.00	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek (15 April 96).

To understand the table above, find today's credit rating on the leftmost column first and then follow along that row to the column that represents the rating at the risk horizon. For example, the figure of 0.02% in the leftmost bottom refers to a 0.02% probability that a CCC/C rated credit will migrate to AAA at the end of one year.

Loss given default

Loss given default, also abbreviated as LGD, is the amount of assets that is lost by a financial firm when a borrower defaults on a loan. Although several methods could be used to calculate the loss given default, there is no widely accepted method of calculating LGD actually. Instead of calculating LGD for each separate loan, most lenders review an entire portfolio of loans and determine LGD based on cumulative losses and exposure.

Besides, as mentioned at the beginning of this chapter, loss given default has a close correlation with recovery rate, and the sum of these two items is equal to one. Their relationship can be expressed as:

$$Loss\ given\ default = 1 - Recovery\ rate, \tag{2.1}$$

In most cases, we estimate LGD by estimating recovery rate first. One of the most widely accepted method to estimate recovery rate is based on seniority ranking of debt. *Tab 2.4* presents statistics for defaulted bond prices.

Tab 2.4: Recovery statistics by seniority class - Par (face value) is $100.00.

Seniority Class	**Carty & Lieberman [96a]**			**Altman & Kishore [96]**		
	Number	**Average**	**Std. Dev.**	**Number**	**Average**	**Std. Dev.**
Senior Secured	115	$53.80	$26.86	85	$57.89	$22.99
Senior Unsecured	278	$51.13	$25.45	221	$47.65	$26.71
Senior Subordinated	196	$38.52	$23.81	177	$34.38	$25.08
Subordinated	226	$32.74	$20.18	214	$31.34	$22.42
Junior Subordinated	9	$17.09	$10.90	-	-	-

Source: Carty & Lieberman [96a] – Moody's Investors Service

As *Tab 2.4* shows, recovery rates may be quite different in different seniority class. For example, when the face value is $100.00, the average value of recovery rates of senior secured debt is $53.80, while the average value of recovery rates of junior subordinated debt is only $17.09.

Exposure at default

Exposure at default, also abbreviated as EAD, is an assessment of the maximum loss exposure a lender is exposed to at any time when default occurs. Because default is often an uncertain event, we had better consider both the current credit exposure and potential changes in the exposure. Therefore, there are three measures of credit exposure: actual exposure, potential exposure, and total exposure.

Actual exposure refers to contract *c* at time *t* as the maximum of zero and the value of the contract at that time:

$$AE(c,t) = max\{0, V(c,t)\}. \tag{2.2}$$

Potential exposure refers to contract *c* at time *t* is the maximum additional amount that will be lost if default occurs at some time τ , not at time *t*:

$$PE(c,t) = max\left\{0, max_{t<\tau<T}\left\{PV_t\left[V(c,\tau)\right] - V(c,t)\right\}\right\}, \tag{2.3}$$

where *T* means the maturity of the contract and $PV_t[*]$ is a function transforming future values to present values at time *t*.

Total exposure is the sum of actual exposure and potential exposure:

$$TE(c,t) = AE(c,t) + PE(c,t). \tag{2.4}$$

Moreover, expected loss (EL), driven by the expected probability of default and the expected recovery rate in default, can be expressed as a simple formula:

$$EL = PD \cdot LGD \cdot EAD. \tag{2.5}$$

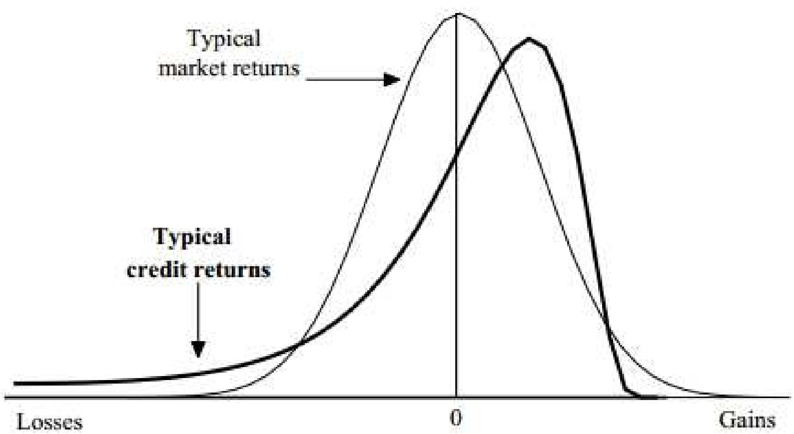
Time horizon

Time horizon is supposed to be longer than the timeframe over which risk-mitigating actions can be taken. There are two alternatives: a constant time horizon and a hold-to-maturity time horizon. The constant time horizon, such as one year, is more suitable for trading desks of banks; while the hold-to-maturity time horizon is almost used by institutions.

2.1.3 Fundamental differences between credit risk and market risk

The fundamental differences between credit risk and market risk can be presented by *Figure 2.2* as below.

Figure 2.2: Comparison of distribution of credit returns and market returns



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 7p.

It is clear in *Figure 2.2* that market returns are relatively symmetric and are well approximately by normal distributions, while credit returns are highly skewed and fat-tailed. Instead of using 95% confidence level, 99% confidence level is usually preferred when estimating credit risk, because the credit risk is still much higher than the market risk at 95% confidence level. Therefore, in order to well understand a credit portfolio's distribution, we usually need more than the two basic statistical measures, namely mean (average) and standard deviation (σ). What's more, the long downside tail of the distribution of credit returns is caused by defaults.

2.1.4 Ratio indicators of credit risk

The following are the most widely used credit risk ratio indicators:

NPL ratio

NPL ratio is the amount of nonperforming loans over total loans, expressed as a percentage. Nonperforming assets are loans that are past due for 90 days or more. The NPL ratio measures a bank's effectiveness in receiving repayments on its loans. NPL ratio is computed by:

$$NPL\ ratio = \frac{Nonperfor\min g\ loans}{Total\ loans\ and\ leases}. \tag{2.6}$$

Note that total loans and leases refer to those loans and leases to customers, not to other financial institutions. Generally, NPL ratio can be used to compare the quality of loan portfolios among banks. A high NPL ratio may indicate high-risk lending policies.

Charge off ratio

Charge off ratio is the ratio of net charge-offs to total loans and leases. Charge-offs are loans that have been declared from the balance sheet and written off. A charge off ratio indicates the performance of loan portfolio of a specific bank, and it is expressed as:

$$Charge\ off\ ratio = \frac{Net\ charge-offs\ of\ loans}{Total\ loans\ and\ leases}. \tag{2.7}$$

Charge off ratio can indicate the quality of loan assets of a bank, and a higher charge off ratio, compared with either the previous periods in one bank or the same time period in other banks, usually calls for attention.

Provisioning rate

Provision for loan losses is an expense that is set aside as an allowance for bad loans. It is a charge against the loan revenues of a bank. Provisioning rate can be computed by:

$$Provisioning\ rate = \frac{Annual\ provision\ for\ loan\ losses}{Total\ loans\ and\ leases\ or\ relative\ to\ equity\ capital}. \tag{2.8}$$

LLA ratio

Loan loss allowance ratio is the ratio of allowance for loan losses over total loans and leases or relative to equity capital. Allowance for loan losses is presented on the balance sheet as an asset account that should be subtracted from gross loans. LLA ratio is expressed as:

$$LLA\ ratio = \frac{Allowance\ for\ loan\ losses}{Total\ loans\ and\ leases\ or\ relative\ to\ equity\ capital}. \tag{2.9}$$

LLA ratio is usually used to predict whether a bank has an adequate loan loss allowance level. However, it is difficult to determine a standard level for LLA ratio among banks, because different banks have different loan portfolios and risk managements.

Coverage ratio

Coverage ratio is the amount of allowance for loan losses over nonperforming loans, and can be expressed as:

$$Coverage\ ratio = \frac{Allowance\ for\ loan\ losses}{Nonperfor\ min\ g\ loans}. \qquad (2.10)$$

It measures the ability of a bank to meet its financial obligations to lenders. Generally speaking, a higher coverage ratio indicates a better ability of a bank to fulfill its financial obligations.

2.2 Market risk

Market risk is the possibility to suffer losses caused by factors that affect the overall performance of the financial markets. It can either be general or risk. General market risk, also named systematic risk, reflects an adverse movement in market prices of all financial instruments; while specific risk reflects an adverse movement in an individual asset's price in day-to-day trading.

2.2.1 Types of market risk

There are four general market risk categories:

- foreign exchange risk,
- interest rate risk,
- equity risk,
- commodity risk.

Foreign exchange risk

Foreign exchange risk arises because of an adverse movement in foreign exchange rates and applies to all exchange rate-related activities. Banks in imperfect hedged positions may be exposed to the foreign exchange risk.

Interest rate risk

Interest rate risk is the potential loss caused by the adverse movement in interest rates. Banks are exposed to interest rate risk in two important ways, and the major difference is between fixed rate assets and liabilities and rate-sensitive assets and liabilities. Fixed rate assets and liabilities refer to constant interest rates during a certain time period and their cash flows do not change unless there is a default or early withdrawal. On the other hand, rate-sensitive assets and liabilities can be re-priced during a certain time period and their cash flows that are associated with rate-sensitive agreements or contracts do change with fluctuations of interest rates.

The interest rate risk is the most important market risk and can be identified as:

$$Interest\ rate\ risk\ ratio = \frac{Interest - sensitive\ assets}{Interest - sensitive\ liabilities}. \tag{2.11}$$

Equity risk

Equity risk is the potential loss due to the adverse movement in the price of equities and applies to the financial instruments, such as derivative products, that use equity prices as part of the valuation. Note that it often refers to equity in companies through the purchase of stocks, but does not refer to real estate or building equity in properties.

Commodity risk

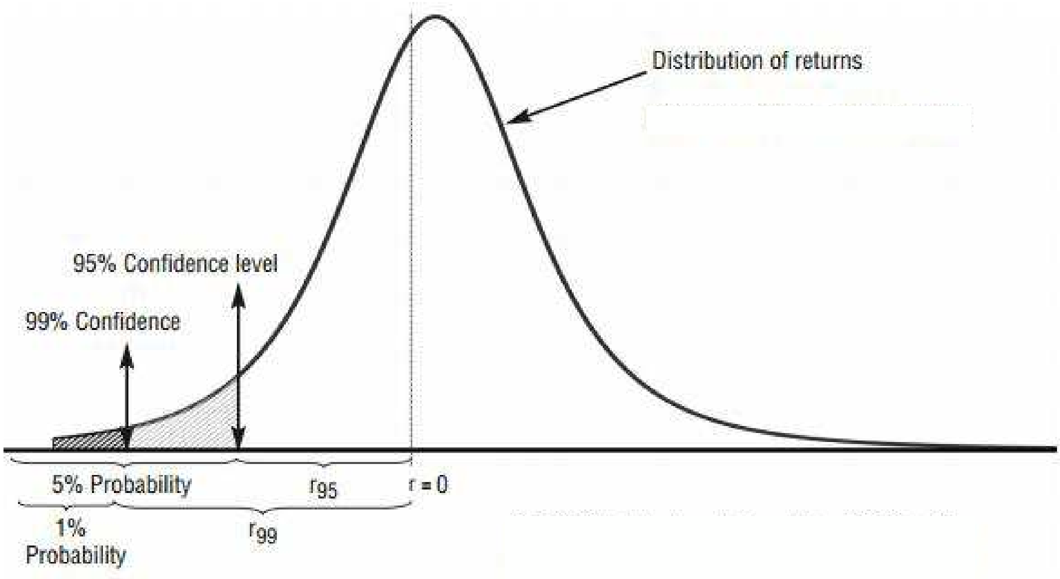
Commodity risk is the potential loss caused by the adverse movement in commodity prices and applies to both commodity and derivative commodity positions. Futures and options are commonly used to hedge against commodity risk. These commodities could be crude oil, gold, grains etc.

2.2.2 Value-at-Risk (VaR)

In order to measure market risk in the portfolios, banks usually use value-at-risk (VaR) approach, which provides the potential loss caused by an adverse market movement at a specific confidence level, usually 95% or 99%, over a given time period. The time period for market risk is typically one day.

It is essential to consider both current positions and distribution of possible return values in the nest time period when calculating VaR. *Figure 2.3* as below presents the return distribution for a portfolio as an example.

Figure 2.3: Graphical interpretation of value-at-risk



Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 170p.

Note that the horizontal *X*-axis refers to the possible gains and losses, and the vertical *Y*-axis refers to the probability of gains or losses. Losses are points to the left of zero, while gains are points to the right of zero. Besides, the sum of the area under the curve must be one.

2.3 Liquidity risk

Liquidity risk is the danger of lacking sufficient cash to meet customer withdrawals, loan demand, and other cash needs. The following are the most widely used liquidity risk ratios:

$$Liquidity\ ratio_1 = \frac{\begin{array}{c}Cash\ and\ due\ from\ balances\\ held\ at\ other\ depository\ institutions\end{array}}{Total\ assets}, \qquad (2.12)$$

$$Liquidity\ ratio_2 = \frac{Cash\ assets\ and\ government\ securities}{Total\ assets}. \quad (2.13)$$

Moreover, there are two more liquidity measurement after Basel III Accord, namely net stable funding ratio (NSFR) and liquidity coverage ratio (LCR).

Net stable funding ratio (NSFR)

Net stable funding ratio estimates the amount of available stable funding relative to the amount of required stable funding. NSFR should be at least 100% and can be expressed as:

$$\begin{matrix} Net\ stable\ funding\ ratio \\ (NSFR) \end{matrix} = \frac{Available\ amount\ of\ stable\ funding}{Required\ amount\ of\ stable\ funding}, \quad (2.14)$$

where *stable funding* includes customer deposits, equity, and long-term wholesale funding. An adequate NSFR can guarantee a stable funding structure.

Liquidity coverage ratio (LCR)

Liquidity coverage ratio can be defined as:

$$\begin{matrix} Liquidity\ coverage\ ratio \\ (LCR) \end{matrix} = \frac{Stock\ of\ high-quality\ liquid\ assets}{\begin{matrix} Total\ net\ cash\ outflows\ over \\ the\ next\ 30\ calender\ days \end{matrix}}, \quad (2.15)$$

where *high-quality liquid assets* include not only the government and public sector entity assets, but also the high-quality corporate and covered bonds.

There will be a bank run when a bank fail to meet depositors' demands, and then depositors will lose confidence and rush to the bank to withdraw funds. It is a bad signal because the bank may have difficulties in obtaining funds in the interbank market and finally a liquidity crisis will occurs. Therefore, in order to manage the liquidity risk more effectively, it is very necessary to distinguish two types of liquidity risk:

- day-to-day liquidity risk,
- liquidity crisis.

Day-to-day liquidity risk

Day-to-day liquidity risk relates to daily withdrawals made by depositors. This type of liquidity risk is relatively easy to be predicted and managed, because there will be only a small percentage of deposits of one single bank being withdrawn on one day. Therefore, very few banks will run out of cash because they can borrow funds from other banks in the interbank markets to cover the shortage of cash easily.

Liquidity crisis

A liquidity crisis, a negative financial situation, happens when depositors' demands are much larger than normal level. It is hard to be predicted, and can be caused by either a lack of confidence in one specific bank, or unexpected cash needs. Generally speaking, banks are forced to borrow funds at a higher interest rate in this situation.

2.4 Operational risk

Operational risk refers to uncertainty caused by inadequate or failed internal processes, people and systems, or by external events. The Basel II Accord considers five broad categories of operational risk events:

- internal process risk,
- people risk,
- systems risk,
- external risk,
- legal risk.

Internal process risk

Internal process risk is caused by the failure of the bank's processes and procedures and inadequate control environment. For example, reports or documents required are not accurate, a teller adds an extra zero to a deposit, and a bank fails to audit recorded transactions etc. Therefore, it is essential to monitor and improve a bank's internal processes to increase its operating efficiency and overall profitability.

People risk

People risk is usually associated with employee errors or frauds. There are several factors that will cause people risk, including high staff turnover, poor management practices, poor staff training, and overreliance on key staff. People risk is a high-frequency operational risk.

Systems risk

Systems risk occurs with computer, technology, and systems failures, given the fact that all banks heavily rely on technology in their everyday activities nowadays. Examples of systems risk include data corruption, programming errors, inadequate project control, system security problems, and so on.

External risk

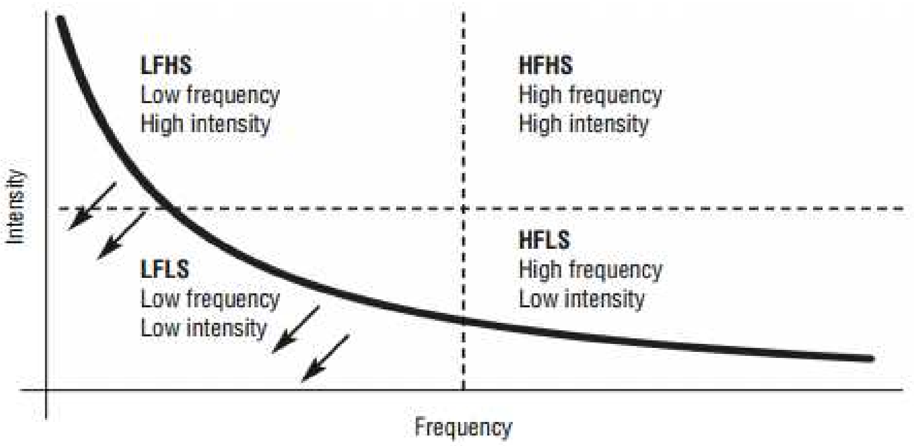
External risk is the risk associated with events having negative impact that are beyond the bank's control, such as external fraud, terrorist attacks, and natural disasters.

Legal risk

Legal risk is caused by the uncertainty of legal action or by the uncertainty of rule and regulation applicability. It generally varies greatly in different counties or in different states because of the differences in laws.

Operational loss events can be divided into two types in general: high-frequency/low impact events and low-frequency/high-impact events. And low-frequency/low-impact events and high-frequency/high-impact events are usually not been concerned with. As presented in *Figure 2.4*, banks are always supposed to make sure that the high-frequency operational risk events are very-low-severity events, and vice versa.

Figure 2.4: Loss intensity and frequency chart of operational risk events



Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 188p.

In order to manage operational risks, it is essential to make sure that high-frequency operational risk events are very-low-severity events, and that high-severity events are very-low-frequency events. Petty fraud and process failures are typical examples of HFLS events, while rogue traders, terrorist attacks, and fires are typical examples of LFHI events.

2.5 Other risk types

Apart from the four typical financial risks, there are some other types of risks worth being mentioned, including regulatory risk, settlement or payment risk, and reputational risk.

Regulatory risk

Regulatory risk is the potential loss arising from the probability that a change made by either the government or a regulatory body in laws and regulations. This change usually has a negative impact on a business or market because of the increased costs of business operations. For example, a policy made by the government requires an increase of the excise duty on tobacco will lead to an increase of the costs of tobacco, and then the prices of tobacco in the market will increase.

Settlement or payment risk

Settlement or payment risk refers to a risk that one party fails to deliver assets or pay money to another party at the time of settlement. This type of risk can be associated with any timing differences in settlement between the two parties. It can be called "Herstatt risk" because of the famous failure of the German bank Herstatt. On Jun 26, 1974, the bank had taken in its foreign-currency receipts in Europe, but had not made any of its U.S. dollar payments when German banking regulators closed the bank down, leaving counter parties with the substantial losses[1].

Reputational risk

Reputational risk is one of microeconomic risks that generally are caused by factors inside the banks. It is a risk or a hidden threat resulting from damages to a bank's reputation. The factors that lead to these kind of damages include the wrong actions of the bank itself, the criminal events the bank is associated with, or even the mistakes made by the bank's joint venture partners.

[1] http://www.investopedia.com/terms/s/settlementrisk.asp.

3 Description of the credit risk management and models

In this chapter, the different categories of models of credit risk management are introduced in simplest version at first, including scoring models, rating systems, and portfolio models. Then we focus on how CreditMetircs™ model works and regulation of capital requirement under Basel I, II, and III respectively.

3.1 Models of credit risk management

During the last decade, the world's largest banks devoted themselves to develop sophisticated systems to model the credit risk in an attempt to help banks quantify, aggregate, and manage risk better and more effectively. There are mainly three categories of models of credit risk management, including scoring models, rating models, and portfolio models. The mechanism used to construct these models will be discussed in detail in the following subchapters.

3.1.1 Scoring models – Altman z-score model

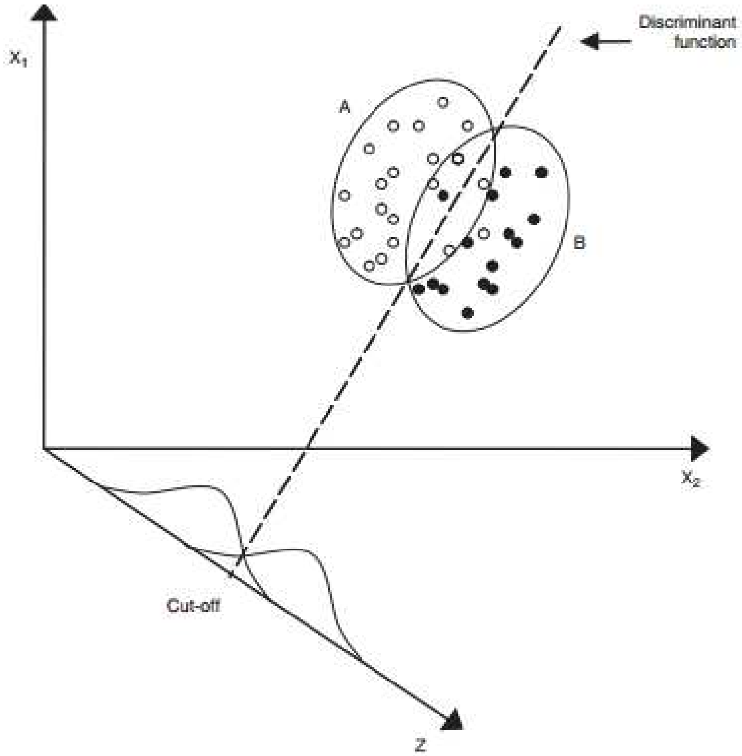
Credit scoring models, a class of statistical models, use the main financial and economic indicators of a borrower to estimate the probability of default of the borrower, discuss the benefits and limitations, and analyze the technical characteristics. The result calculated by a scoring model is expressed as a numerical score and is an index of creditworthiness used to estimate the probability of default of the borrower.

Among different categories of scoring models, the most basic category of scoring models is linear discriminant analysis (LDA), which was studied by Fisher as early as 1936, and is based on a deductive approach designed to identify the economic causes of default. *Basically, discriminant analysis is a classification technique which uses data obtained from a sample of companies to draw a boundary that separates the group of reliable ones from the group of insolvent ones*[2].

[2] ANDREA, S. and ANDREA, R. *Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies*. Wiley Finance, 2007. 287p.

The following *Figure 3.1* presents the Fisher model in a simplified case in which reliable (A) and insolvent companies (B) are described by only two variables, namely x_1 and x_2. The score generated by combing the two original variables in shown on the z axis.

Figure 3.1: Graphic representation of linear discriminant analysis



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 288p.

Linear discriminant analysis constructs the score z as a linear combination of the independent variables, x_1 and x_2. The cut-off point is the point at which the bank decides whether or not grants a loan to the company. Given n independent variables, the score can be computed in more general terms as:

$$z = \sum_{j=1}^{n} \gamma_j x_j \ , \tag{3.1}$$

$$z_i = \sum_{j=1}^{n} \gamma_j x_{i,j} \ . \tag{3.2}$$

Note that the coefficients γ_j in this linear combination are chosen to obtain a score z which can discriminate between abnormal and healthy companies as clearly as possible.

The most famous discriminant score applied to credit risk is Altman's Z-score, developed by Edward Altman in 1968 for listed U.S. companies and now offered as Zeta Services Inc. commercially. The greater the z score, the better a company's quality. It is a function of five independent variables and can be formulated as:

$$z_i = 1.2 \cdot x_{i,1} + 1.4 \cdot x_{i,2} + 3.3 \cdot x_{i,3} + 0.6 \cdot x_{i,4} + 1.0 \cdot x_{i,5}, \qquad (3.3)$$

where: x_1 = working capital/total assets, x_2 = retained profits/total assets, x_3 = earnings before interest and tax/total assets, x_4 = market value of equity/book value of total liabilities, x_5 = turnover/total assets.

Altman sets the cut-off point at a value of 1.81, which was obtained as the average between the mean value of z for a sample of healthy companies and that for a sample of insolvent companies. Practically speaking, if a company's corresponding z score is lower than 1.81, then it is regarded as too risky according to Altman's z-score model.

3.1.2 Rating systems

When talking about rating systems nowadays, they generally consist of both qualitative analyses and quantitative models. Qualitative analyses are used by international credit rating agencies such as Moody's, Standard & Poor's, and Fitch; while quantitative models could be discriminant analysis for instance to contribute to the company's financial indicators. There are three steps of a rating system:

- rating assignment,
- rating quantification,
- rating validation.

Rating assignment

Rating assignment includes both the assignment of agency ratings and the assignment of internal ratings as mentioned in the previous chapter. The medium financial ratios are used to assess the financial situation of a company. The higher the rating, the higher the profitability and capitalization.

Rating quantification

Once being assigned a rating, it is necessary to obtain the PDs of the borrowers for risk measurement purposes. Three approaches may be used to solve this problem:

- the statistical approach – based on the score obtained by the scoring model,
- the actuarial approach – based on the actual default frequencies,
- the mapping approach – a link between internal ratings and external ratings.

Among these three possible approaches, the actuarial approach is the most widely used by both banks and rating agencies, which publish statistics periodically on the defaults recorded in earlier years. Specifically speaking, if the past data show that 3% of the borrowers who are assigned to class BB tend to default in one year, then a PD of 3% will be assigned to all borrowers in class BB now.

The marginal default rate for year t can be computed by:

$$d_t' = \frac{D_t}{N_t}, \tag{3.4}$$

where D_t denotes the number of defaults recorded in year t, and N_t denotes the number of issuers present at the start of year t. Equivalently, the marginal survival rate s_t' can be expresses as:

$$s_t' = \frac{N_t - D_t}{N_t} = 1 - d_t'. \tag{3.5}$$

Then the cumulative default rate and the cumulative survival rate between 0 and T can be formulated respectively as:

$$d_T = \frac{\sum_{t=1}^{T} D_t}{N_1}, \tag{3.6}$$

$$s_T = 1 - p_T = \frac{N_1 - \sum_{t=1}^{T} D_t}{N_1}. \tag{3.7}$$

Moreover, given the equation that $N_{t+1} = N_t - D_t$, so s_t and d_T can be rewritten as:

$$s_T = \prod_{t=1}^{T} s_t^{'}, \tag{3.8}$$

$$d_T = 1 - s_T = 1 - \prod_{t=1}^{T} \left(1 - d_t^{'}\right). \tag{3.9}$$

Rating validation

One useful method of validating rating assignments is associated with contingency tables, which includes four quadrants:

Tab 3.2: Example of a contingency table

		Performing	Defaulting
Rating by model	Low-risk (“pass”)	Correct valuation (N_1 cases)	Type I errors (N_2 cases)
	High-risk (“fail”)	Type II errors (N_3 cases)	Correct evaluations (N_4 cases)

Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders’ Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 389p.

- N_1 = the number of companies correctly rated as "healthy",
- N_2 = the number of companies incorrectly rated as healthy,
- N_3 = the number of companies incorrectly rated as being too risky,
- N_4 = the number of companies correctly rated as high-risk.

3.1.3 Portfolio models

Apart from scoring models and rating systems mentioned above, portfolios models, developed during the second half of the Nineties, also can be used to quantify the unexpected loss effectively on a portfolio of credit exposures. Portfolio models are designed to *determine the maximum loss a credit portfolio can face during a predetermined time horizon with a certain confidence level (that is, the so-called "maximum probable loss")*[3]. There are four portfolio models particularly:

- KMV (1993),
- CreditRisk+™ (1997),
- CreditPortfolioView™ (1997),
- CreditMetrics™ (1997).

Credit risk models are used to estimate the economic capital required to cover the risks that associated with the banks' lending activities. In order to define the credit losses and then quantify the credit risk, there are two basic approaches:

- Mark to market (MTM): the borrower is defined in any grades at the end of the risk horizon, including default and migration, then the risk arises when the rating grade of the borrower migrates from a higher rate to a lower rate;
- Default model: a model used by financial institutions to distinguish the borrower with only two states, namely default or survival, at the end of the risk horizon, then the risk arises from the default.

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[3] ANDREA, S. and ANDREA, R. *Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies*. Wiley Finance, 2007. 401p.

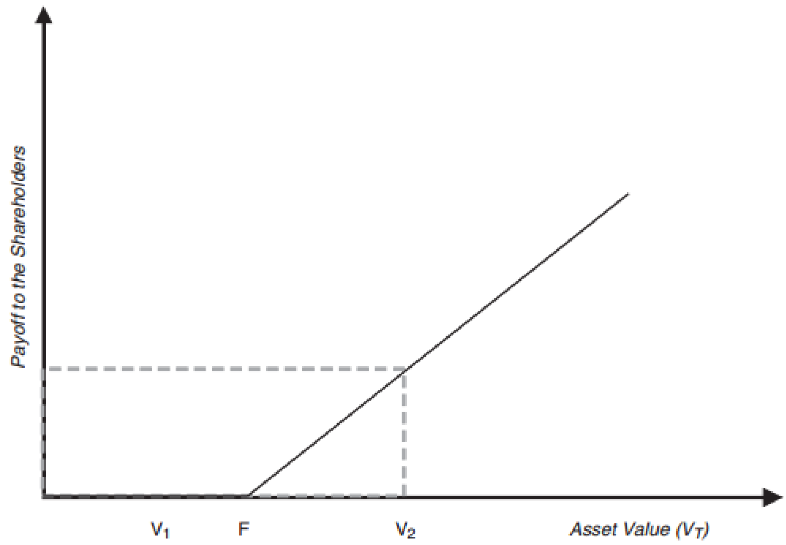
KMV

KMV model is developed by the California-based company KMV, and the acronym KMV comes from the last names of the three founding partners, namely Steven Kealhofer, John Andrew McQuown, and Oldrich Vasicek. The model claims that *the value of equity (E) is equal to the value of a call option on the market value of the company's assets, with a maturity equal to the residual life of its debt (T) and a strike price equal to the nominal repayment value of the debt (F)*[4]. *Tab 3.3* and *Figure 3.1* below present how the two positions produce the same result at maturity (T) respectively.

Tab 3.3: Matrix of payoffs as a shareholder or for the purchase of a call option on asset value with a strike price of F

	Payoff at time 0	Payoff at T	
		if $V_T < F$	if $V_T > F$
Shareholder	$-E_0$	0	$(V_T - F)$
Purchase of a call option	$-C_0$	0	$(V_T - F)$

Figure 3.1: Shareholder payoff profile



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 330p.

[4] ANDREA, S. and ANDREA, R. *Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies*. Wiley Finance, 2007. 332p.

When V_T is lower than F, the company is insolvent and the remaining assets have to be used to repay the debt entirely, then the shareholders will get nothing. Conversely, when V_T is greater than F, the company is profitable and the difference between V_T and F is the amount the shareholders will gain.

Generally speaking, KMV model requires three steps to estimate a company's probability of default indirectly:

- Compute the default point (DP), which is the sum of all short-term debt (STD) and 50% of long-term debt (LTD), as:

$$DP = STD + \frac{1}{2} LTD. \tag{3.10}$$

- Estimate the distance of default (DD), which is the difference between current value of assets and the default point. This risk index can be formulated as:

$$DD = \frac{V_0 - DP}{V_0 \cdot \sigma_V}. \tag{3.11}$$

- Convert the distance of default into a probability of default based on a fairly precise empirical correlation, which is based on the actual past evidence. The link between DDs and PDs is called expected default frequency (EDF).

CreditRisk+™

CreditRisk+™ model is a statistical model proposed by Credit Suisse Financial Products (CSFP), a London-based subsidiary of the Swiss banking group Credit Suisse, which is based on the actuarial mathematical models derived from the insurance industry. CreditRisk+™ model considers that *default rates as continuous random variables and incorporates the volatility of default rates in order to capture the uncertainty in the level of default rates*[5]. This model is widely applied in the

[5] http://www.csfb.com/institutional/research/assets/creditrisk.pdf.

management of some traditional banking portfolios, including consumer loans, mortgages, and loans to small or medium enterprises.

CreditRisk+™ model describes the probability distribution of n defaults over a given risk horizon, typically one year, through an essential tool of actuarial mathematics named the Poisson distribution. So the probability can be computed as:

$$p(n)=\frac{e^{-\mu}\cdot\mu^{n}}{n!}, \tag{3.12}$$

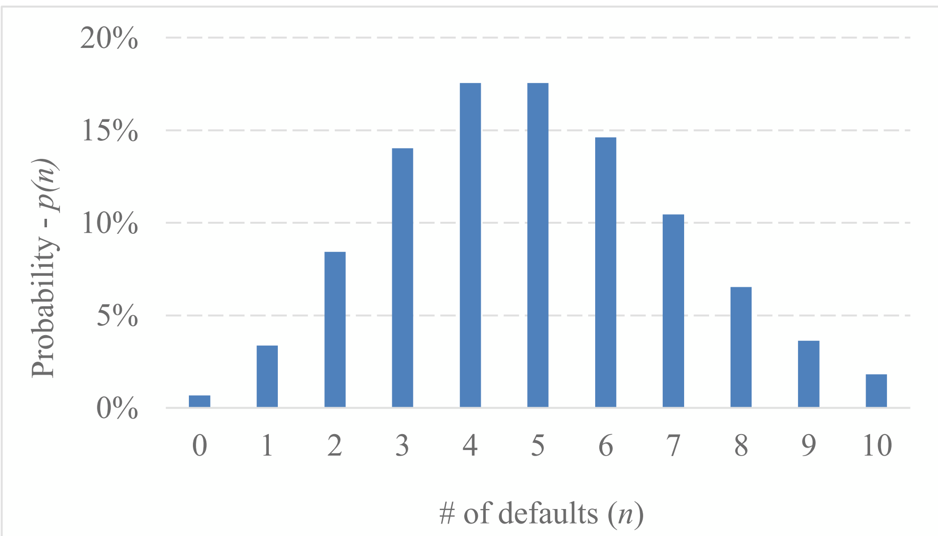
where μ is expected number of defaults, representing the sum of all the PDs of the customers in the portfolio.

Let us take a specific example now. Assume that there are 500 clients in a bank, and each client's PD equals to 1%, then the value of μ will be 5. The probability that no defaults occur, which means n equals to zero, can be calculated as:

$$p(0)=\frac{e^{-5}\cdot 5^{0}}{0!}=0.67\%.$$

Figure 3.2 below shows values of $p(n)$ computed with equation (3.12) for n between 0 and 10 in the same manner.

Figure 3.2: An example of Poisson distribution



It is obvious that the probability distribution in this specific case is abnormal distribution and right-tailed. In other words, when values of *n* increases, the probability gradually decreases near to zero after the peak point.

Note that CreditRisk+™ model assumes that the PDs of borrowers and the recovery rates on loans have already been known, therefore, it does not produce the estimation of PDs or explain the process that causes the default of a company.

CreditPortfolioView™

CreditPortfolioView™ (CPV™) model is a model based on the observation that credit cycles depend on the current phase of economic cycle, developed in 1997 by Tom Wilson in the consulting firm McKinsey. The migration rates toward higher classes tend to be more frequent and the migration rates toward lower classes and defaults tend to decline during phases of economic growth, while the opposite situations occur during phases of recessions. Therefore, CPV™ model *proposes to link the probabilities of migration and default to macroeconomic variables such as interest rate levels, the employment rate, real GDP growth and the savings rate, thus "conditioning them" to the state of the economic cycle*[6].

Given the assumption that the probability of default p_{jt} at time *t* of a group or segment *j* of companies based on macroeconomic factors varies with the economic cycle, the logit function can be formulated as:

$$p_{jt} = \frac{1}{1+e^{-y_{j,t}}}, \tag{3.13}$$

where $y_{j,t}$ is the value of a health index at time *t* of a group or segment *j*. $y_{j,t}$ index is a linear combination of several selected macroeconomic factors and therefore can be estimated based on the historical data.

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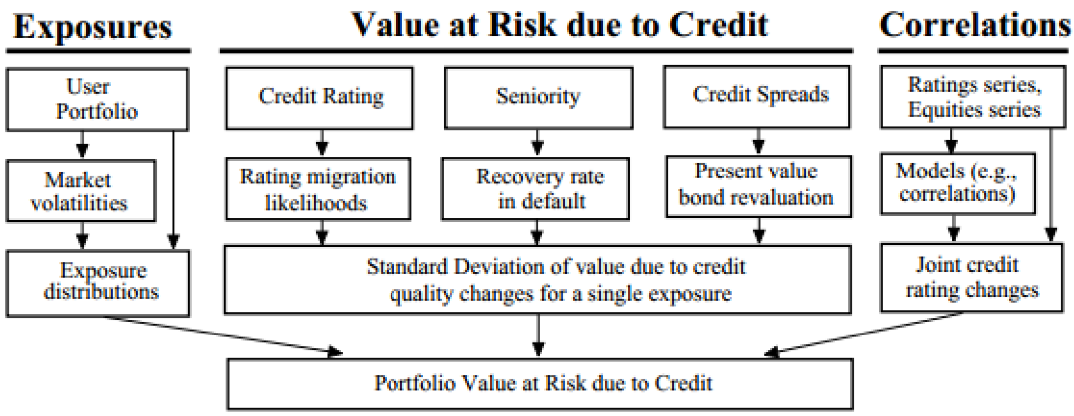
[6] ANDREA, S. and ANDREA, R. *Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies*. Wiley Finance, 2007. 426p.

3.2 Description of CreditMetrics™

CreditMetrics™ model, proposed by the U.S. bank J. P. Morgan originally, is a tool for estimating the distribution of changes in the market value of a portfolio of credit exposures based on the data for migration rates, default rates, and spreads of borrowers. Then it is possible to estimate the expected loss (EL) and the unexpected loss (UL) according to that distribution. *Figure 3.3* below presents a step-by-step introduction of CreditMetrics™ model.

Figure 3.3: Basic framework of CreditMetrics™ model



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 41p.

In this subchapter, CreditMetrics™ model will be described in detail from three main parts, including risk management framework, credit quality correlation, and interpretation and application of results.

Value at risk (VaR) represents the maximum potential losses at a given confidence level, usually 99% but more frequently 99.5% or even 99.9%, over a specific time interval. It can be interpreted in two ways:

a) losses from the portfolio of debt assets $\left(-\Delta\tilde{\Pi}\right)$ set at a significance level of α, which is greater than the predetermined value losses (VaR), can be expressed as:

$$Pr\left(-\Delta\tilde{\Pi} \geq VaR\right) = \alpha \, ; \tag{3.14}$$

b) profit from the portfolio of debt assets $\left(\Delta\tilde{\Pi}\right)$ set at a significance level of α, which is less than the predetermined value gains (-VaR), can be expressed as:

$$Pr\left(\Delta\tilde{\Pi} \leq -VaR\right) = \alpha \, . \tag{3.15}$$

VaR calculation can be performed by using the Monte Carlo method, which is based on a large number of simulations of the development of the portfolio assets. The main task of this model is to determine the probability distribution of the increase in the value of the portfolio assets $\left(\Delta\tilde{\Pi}\right)$ at a given significance level of α, which can be written as follows:

$$\Delta\tilde{\Pi} = \tilde{V}_P^T - V_P^t = \sum_n \tilde{V}_{n,J,T} \cdot x_n - \sum_n V_{n,i,t} \cdot x_n \, . \tag{3.16}$$

where $\tilde{V}_P^T$ is the default value of the portfolio, V_P^t is the predicted value of the portfolio, $V_{n,i,t}$ is the value of n-th asset with i-th rating category in the portfolio, x_n is the amount of n-th asset with i-th rating category in the portfolio, $\tilde{V}_{n,J,T}$ is the value of n-th asset with i-th rating category in the portfolio at the end of a predetermined time horizon T. It is usually a one-year time horizon and the value is derived from a grade at the end of the time horizon.

Economic capital, which is the difference between unexpected losses and expected losses, represents an estimation of the amount of the required capital to maintain unexpected losses at a specific significance level. It can be computed when using the probability distribution of portfolio gains as:

$$Economic\ capital = VaR_\alpha - E\left(-\Delta\tilde{\Pi}\right), \tag{3.17}$$

where $E\left(-\Delta\tilde{\Pi}\right)$ is the mean value of gains, VaR_{α} is determined so that the simulated values of the portfolio returns are ranked according to the order and the value of VaR at a specific significance level will be equal to n-th worst. But with the opposite sign, it is possible to use the following formula mathematically:

$$n = \alpha \cdot N, \tag{3.18}$$

where n is the ordered number of the experiment and N is the number of scenarios.

3.2.1 Risk management framework

There are mainly four steps to calculate the credit risk for a portfolio by using CreditMetrics™ model as illustrated in *Figure 3.3* above:

- Step 1: Credit rating migration
- Step 2: Calculation of the present value of a bond
- Step 3: Calculation of the discount rate
- Step 4: Credit risk estimation

Step 1: Credit rating migration

CreditMetrics™ model assumes that each exposure has been assigned a rating no matter by an external rating agency or by a bank itself. Besides, those rating grades are indicative of the default and migration probabilities for the subsequent year. Then, risk comes. It is important to estimate both the likelihood of default and the chance of migrating to any possible credit quality state at the same risk horizon. *Tab 2.4* is a typical example. It is necessary to specify both the default likelihood and the likelihoods that companies in one category migrate to other no matter how many rating categories or how these categories are constructed.

Step 2: Calculation of the present value of a bond

In this step, values are determined at the risk horizon and there are usually eight revaluations in simple one-bond case because value should be calculated separately for each migration state. Moreover, the eight valuations can be divided into two categories – one is in the event of default, and other is in the event of upgrades or downgrades.

In the first case (in the event of a default), the recovery rate is estimated depended on the seniority classification of the debt. *Tab 3.4* summarizes the RRs in the state of default:

Tab 3.4: Recovery rates by seniority class (% of face value, i.e., "par")

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Carty & Lieberman [96a] – Moody's Investors Services

For instance, if a BBB bond is senior unsecured, its mean value in default is supposed to be 51.13% of its face value, and its standard deviation of the recovery rate is 25.45%.

In the second case (in the event of upgrades or downgrades), the change in credit spread is estimated based on a straightforward present value bond revaluation. The present value (PV) of a bond can be calculated as:

$$PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \cdots + \frac{C+M}{(1+i)^n}, \tag{3.19}$$

where C = coupon payment, n = number of payments, i = interest rate, or required yield, M = value at maturity, or par value, and $C+M$ = nominal value.

In order to well understand, let us illustrate this case in detail with the help of another example. Assume that a BBB bond has a five-year maturity, a face value of \$100, and pays annual coupons at the rate of 5%. Therefore, the bond pays \$5 each at the end of the next four years. *Tab 3.5* below presents the forward zero cures for each rating category.

Tab 3.5: Example one-year forward zero curves by credit rating category (%)

Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 27p.

If this BBB bond upgrades to single-A for instance, the value of the bond can be formulated as:

$$V = 5 + \frac{5}{\left(1+3.72\%\right)} + \frac{5}{\left(1+4.32\%\right)^2} + \frac{5}{\left(1+4.93\%\right)^3} + \frac{105}{\left(1+5.32\%\right)^4} = 104.08\,.$$

After completing the calculations in the same manner as above, the values for different rating categories can be obtained in *Tab 3.6*:

Tab 3.6: Possible one-year forward values for a BBB bond plus coupon

Year-end rating	Value ($)
AAA	104.78
AA	104.60
A	104.08
BBB	103.00
BB	97.59
B	93.76
CCC	79.72
Default	51.13

Step 3: Calculation of the discount rate

The discount rate accounts for the time value of money, which is derived from the forward zero curve for each specific rating category to discount the cash flows. There is a need to add one line that represents the transition probability of default rating to another. Unfortunately, this situation cannot happen realistically and therefore we usually constitute a row of zeros. If a company is in default, the last column will be equal to one. The result of this adjustment is a matrix T that can be computed by:

$$T = \begin{vmatrix} T_V & t_d \\ 0 & 1 \end{vmatrix}. \tag{3.20}$$

Then it is possible to specify a two-year transition matrix, which is calculated as the product of T and T, as follows:

$$T^2 = T \cdot T = \begin{vmatrix} T_V^2 & \left(1 + T_V\right) \cdot t_d \\ 0 & 1 \end{vmatrix}. \tag{3.21}$$

Similarly, the n-year transition matrix can be formulated as:

$$T^n = \begin{vmatrix} T_V & \sum_{t=0}^{n-1} T_V^i t_d \\ 0 & 1 \end{vmatrix}, \tag{3.22}$$

where T^n denotes the probability of default of the company during n years for all rating categories.

Now the discount rates can be determined for different rating categories and years. The calculation of these rates is based on the risk-free rate and the implicit expectations theory, which can be expressed as:

$$f_t = \frac{\left(1 + r_t\right)^t}{\left(1 + r_{t-1}\right)^{t-1}} - 1, \tag{3.23}$$

where r_t is the risk-free rate at which it may be selected, such as PRIBOR, LIBOR, EURIBOR, 2W REPO value or IRS (interest rate swap).

Relationship for one-year interest rate can be expressed as:

$$\left(1+r_1^i\right)\cdot\left(1+p_1^i\right)+p_1^i\cdot RR=1+r_i^F \, , \tag{3.24}$$

where r_1^i is the desired rate of a company with a rating in one year, p_1^i is the probability of default, and r_i^F is a one-year risk-free rate.

Then it is necessary to estimate a three-year interest rate due to the relationship:

$$p_1^i\cdot RR\cdot\frac{\left(1+r_1^F\right)^2}{\left(1+r_1^F\right)}+\left(p_2^i-p_1^i\right)\cdot RR+\left(1+r_2^i\right)\cdot\left(1-p_2^i\right)=\left(1+r_2^F\right)^2 , \tag{3.25}$$

where p_1^i and p_2^i are the probability of default in the first and second year respectively.

The mathematical adjustment can be obtained by the equation for the two-year interest rate as follows:

$$r_2^i=\sqrt{\frac{\left(1+r_2^F\right)^2-p_1^i\cdot RR\cdot\frac{\left(1+r_1^F\right)^2}{\left(1+r_1^F\right)}-\left(p_2^i-p_1^i\right)\cdot RR}{\left(1-p_2^i\right)}}=-1 . \tag{3.26}$$

The last equation can, if it is necessary, be adjusted for calculating the *n*-year interest rate, which is computed by:

$$r_n^i=\sqrt{\frac{\left(1+r_n^F\right)^n-RR\cdot\sum_{j=1}^{n}\left[p_{j-1}^i\cdot\frac{\left(1+r_n^F\right)^n}{\left(1+r_{j-1}^F\right)^{j-1}}+\left(p_j^i-p_{j-1}^i\right)\right]}{\left(1-p_n^i\right)}}=-1 . \tag{3.27}$$

Step 4: Credit risk estimation

The last step is to estimate the volatility or standard deviation of value due to credit changes for a single exposure. According to what we have already obtained from previous two steps, it is able to obtain the likelihoods of all possible outcomes and the distribution of value with each possible outcome, which are shown in *Tab 3.7*:

Tab 3.7: Calculating volatility in value due to credit quality changes

Year-end rating	Probability of state (%)	New bond value plus coupon ($)	Probability weighted value ($)	Difference of value from mean ($)	Probability weighted difference squared
AAA	0.02	104.78	0.02	2.23	0.0010
AA	0.33	104.60	0.35	2.05	0.0139
A	5.95	104.08	6.19	1.53	0.1394
BBB	86.93	103.00	89.53	0.45	0.1726
BB	5.30	97.59	5.17	-4.96	1.3030
B	1.17	93.76	1.10	-8.80	0.9051
CCC	0.12	79.72	0.10	-22.83	0.6253
Default	0.18	51.13	0.09	-51.42	4.7594
	Mean =	$102.55		Variance =	7.9197
				Standard deviation =	$2.81

Generally, there are two useful measures of credit risk, namely standard deviation and percentile level. Let us consider the calculation of the standard deviation at first. In *Tab 3.7*, the mean μ, the probability-weighted average of the values of all possible rating categories, and the standard deviation σ, the volatility of value, can be computed respectively as:

$$\mu = \sum_{i=1}^{s} p_i \mu_i \ , \tag{3.28}$$

$$\sigma = \sqrt{\sum_{i=1}^{s} p_i \mu_i^2 - \mu^2} \ , \tag{3.29}$$

where p_i = probability of being in a certain state, μ_i = new bond value plus coupon, and s = number of categories.

The second useful measure is the percentile level. The value at risk can be computed associated with a selected confidence level, usually 95% or 99%, to narrow the distribution of value changes. It is necessary to rewrite the probability weighed value in an ascending order, and the order of probability of state next to it are ought to change accordingly.

Tab 3.8: Values and cumulative probabilities

Year-end rating	Difference of value from mean ($)	Probability of state (%)	Cumulative probability (%)	New bond value plus coupon ($)
Default	-51.42	0.18	0.18	51.13
CCC	-22.83	0.12	0.30	79.72
B	-8.80	1.17	1.47	93.76
BB	-4.96	5.30	6.77	97.59
BBB	0.45	86.93	93.70	103.00
A	1.53	5.95	99.65	104.08
AA	2.05	0.33	99.98	104.60
AAA	2.23	0.02	100.00	104.78

In *Tab 3.8* above, the VaR at 99% confidence level can be found at a loss value of -8.80 by isolating at least 1% of the worst cases, because the cumulative probability in this row is 0.30%, which is lower than 1%, while that is 1.47% in the next row, which is greater than 1%. Then, the value, which is equal to $93.76, is the 1% percentile level value. This is $8.79 below the mean value. In a similar way, if the confidence level is 95%, the value is equal to $97.59 and is $4.96 below the mean value.

Let us now extend the credit risk calculation from a single exposure to the multiple exposures. For clarify, an example portfolio consisting of two specific bonds are chosen to be considered. One bond is a BBB-rated bond, senior unsecured, 5% annual coupon, and five-year maturity. Other one is an A-rated bond, senior unsecured, 4% annual coupon, and three-year maturity. Therefore, the joint likelihoods of credit quality migrations should be estimated because we have to consider the effects of non-zero credit quality correlations.

Tab 3.9 shows the probability of joint migration of two obligors with ratings A and BBB, assuming a 0.30 asset correlation. We postpone the discussion of credit quality correlation to next subchapter.

Tab 3.9: Joint migration probabilities with 0.30 asset correlation (%)

Obligor 1 (BBB)		Obligor 2 (single-A)							
		AAA	AA	A	BBB	BB	B	CCC	D
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.04	0.29	0.00	0.00	0.00	0.00	0.00
A	5.95	0.02	0.39	5.44	0.08	0.01	0.00	0.00	0.00
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0.00	0.02	4.47	0.64	0.11	0.04	0.00	0.01
B	1.17	0.00	0.00	0.92	0.18	0.04	0.02	0.00	0.00
CCC	0.12	0.00	0.00	0.09	0.02	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.13	0.04	0.01	0.00	0.00	0.00

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 38p.

It is simple to calculate the possible one-year forward values for a single-A bond plus coupon in the similar way as *Tab 3.4*, then we obtain the results shown in *Tab 3.10* as below:

Tab 3.10: All possible 64 year-end values for a two-bond portfolio ($)

Obligor 1 (BBB)		Obligor 2 (single-A)							
		AAA	AA	A	BBB	BB	B	CCC	D
		103.70	103.61	103.42	102.77	100.31	98.58	86.09	51.13
AAA	104.78	208.48	208.38	208.20	207.55	205.09	203.35	190.86	155.91
AA	104.60	208.30	208.21	208.02	207.37	204.91	203.18	190.69	155.73
A	104.08	207.78	207.69	207.50	206.85	204.40	202.66	190.17	155.21
BBB	103.00	206.70	206.60	206.42	205.77	203.31	201.57	189.08	154.13
BB	97.59	201.29	201.20	201.01	200.36	197.91	196.17	183.68	148.72
B	93.76	197.46	197.36	197.18	196.52	194.07	192.33	179.84	144.89
CCC	79.72	183.43	183.33	183.15	182.49	180.04	178.30	165.81	130.85
D	51.13	154.83	154.74	154.55	153.90	151.44	149.71	137.22	102.26

As far as the portfolio standard deviation is concerned, we use the equations (3.28) and (3.29) in the similar way in the single exposure case. The mean μ and the standard deviation σ for a two-bond portfolio can be calculated as:

$$\mu = \sum_{i=1}^{s=64} p_i \mu_i = \$206.15 \,,$$

$$\sigma = \sqrt{\sum_{i=1}^{s=64} p_i \mu_i^{\,2} - \mu^2} = \$3.10 \,.$$

If we concern the percentile level and the confidence level is 99%, then the likelihoods of all the values should be less than the sum to 1%. As shown in *Tab 3.7* and *Tab 3.10*, it is simple to obtain the number of \$197.18, which is \$8.97 below the mean value.

Marginal risk

Here we finished the discussion of the credit risk measures for both a single exposure and multiple exposures, then we will introduce the marginal risk, an additional amount of risk associated with a new credit tool adds to an existing portfolio. Marginal risk can be calculated by using both the standard deviation and the percentile level.

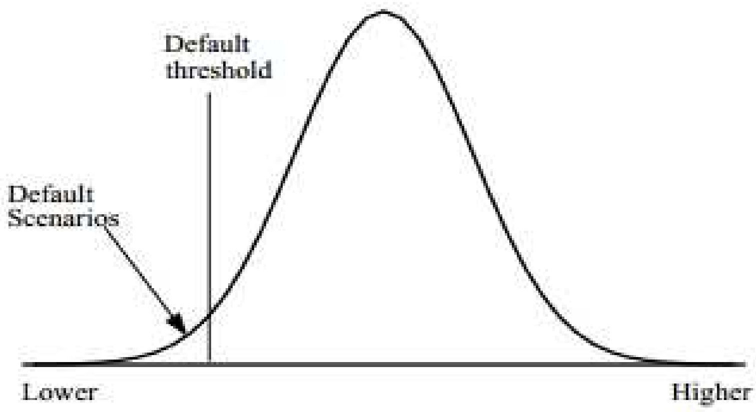
When using the standard deviation as the credit risk measure, the standard deviation of BBB-rated bond is \$2.81, while the portfolio standard deviation is \$3.10. Thus, the marginal standard deviation of the added single-A bond is equal to \$0.29, which is the difference between \$3.10 and \$2.81. Moreover, the marginal standard deviation is lower than the standard deviation of single-A rated bond, because the individual bonds are not perfectly correlated.

When using the percentile level and the confidence level is 99% again, the BBB-rated bond has a mean value of \$102.55 and a 1% percentile level value of \$93.76 as shown in *Tab 3.6*, while the two-bond portfolio has a mean value of \$206.15 and a 1% percentile level value of \$197.18 as shown in *Tab 3.8*. Then the marginal risk of the added single-A rated bond is equal to \$0.18, which is the difference between \$8.97 and \$8.79.

3.2.2 Credit quality correlation

Generally, it is too simplistic and unrealistic to assume a zero correlation because *rating changes and defaults of companies are partly the result of common factors, such as the economic cycle, shifts in interest rates, changes in commodity prices, and so forth*[7]. The asset correlation is equal to 0.30 in *Tab 3.7* for instance. CreditMetrics™ model requires the joint likelihood of credit movements among obligors, which means estimating the credit quality correlation parameters. Let us regard default as a function of the company value as *Figure 3.4* below:

Figure 3.4: Model of company value and its default threshold



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 37p.

The distribution of company value is random. As shown in *Figure 3.4*, when the value of the company starts to decrease until it is less than the amount of liabilities outstanding – the default threshold – the company will have difficulty in meeting its obligations and it will move towards bankrupt finally.

Now extend the model above to contain rating changes. There exist credit rating upgrade thresholds and downgrade thresholds as well as the default thresholds. In *Figure 3.5*, the company value and generalized credit quality threshold are illustrated.

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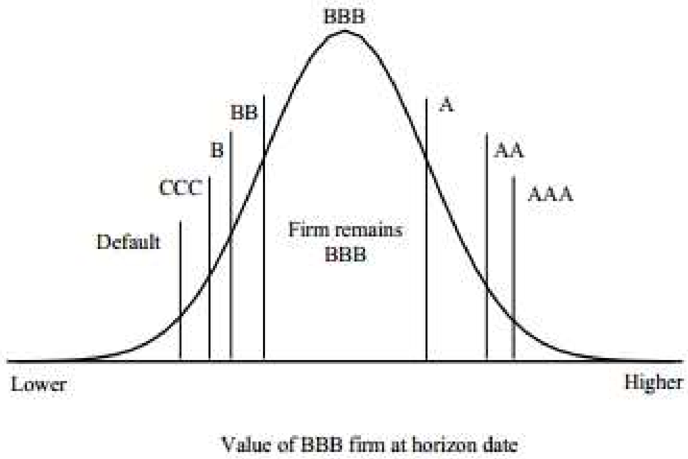
[7] ANDREA, S. and ANDREA, R. *Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies*. Wiley Finance, 2007. 413p.

Figure 3.5: Model of company value and generalized credit quality threshold



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 37p.

Before determining the actual correlation, there is a need to conduct several partial calculations as below.

The discrete returns of shares can be formulated as:

$$R_i = \frac{P_t - P_{t-1}}{P_{t-1}} , \tag{3.30}$$

where R_i is the return of the asset, P_t is the value of the asset at time *t*, and P_{t-1} is the value of the asset at time *t-1*.

The expected return of the *i*-th asset can be computed by:

$$E\left(R_i\right) = \frac{1}{T} \cdot \sum_{t=1}^{T} R_i , \tag{3.31}$$

where $E\left(R_i\right)$ denotes the mean value of returns of assets and *T* denotes the number of observations.

The expected return of the portfolio is a weighted average of the expected return of every single asset, which can be mathematically expressed as:

$$E(R_P)=\sum_{i=1}^{N} E(R_i)\cdot w_i=\vec{w}^T\cdot E(\vec{R}), \tag{3.32}$$

where w_i is the weight of the i-th asset in the portfolio, N is the number of assets in the portfolio, $\vec{w}^T$ is transposed vector variables, and $E(\vec{R})$ is the vector of expected returns of assets.

The variance of the asset is determined as the average of the sum of squared deviations from the expected return of the asset, which can be expressed as:

$$\sigma^2(R_i)=\frac{1}{T}\cdot\sum_{t=1}^{T}\left[R_{i,t}-E(R_i)\right]^2. \tag{3.33}$$

The variance of the overall portfolio can be expressed as:

$$\sigma^2(R_P)=\sum_{i=1}^{N}\sum_{j=1}^{N} w_i\cdot w_j\cdot cov(R_i;R_j)=\vec{w}^T\cdot C\cdot\vec{w}, \tag{3.34}$$

where w_i is the weight of i-th asset in the portfolio, w_j is the weight of j-th asset in the portfolio, σ_{ij} is the covariance of the returns of two assets, and C is the covariance matrix.

Classically, the asset correlation among the assets of a portfolio could by in the range of 20% to 35%[8]. The covariance of the returns of two assets, R_i and R_j, and the correlation coefficient of the returns of these two assets can be given by the following formulas:

$$\begin{aligned}\sigma_{ij}&=E\left[R_{i,t}-E(R_i)\right]\cdot\left[R_{j,t}-E(R_j)\right]\\&=\frac{1}{T}\cdot\sum_{t=1}^{T}\left[R_{i,t}-E(R_i)\right]\cdot\left[R_{j,t}-E(R_j)\right]'\end{aligned} \tag{3.35}$$

[8] This is based on conversations with Patrick H. McAllister in 1994 when he was an Economist at the Board of Governors of the Federal Reserve System.

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \cdot \sigma_j}. \tag{3.36}$$

Correlation between individual issuers can be determined by the matrix. Firstly, it is necessary to establish the correlation matrix *C(m+n; m+n)*, which can be given by:

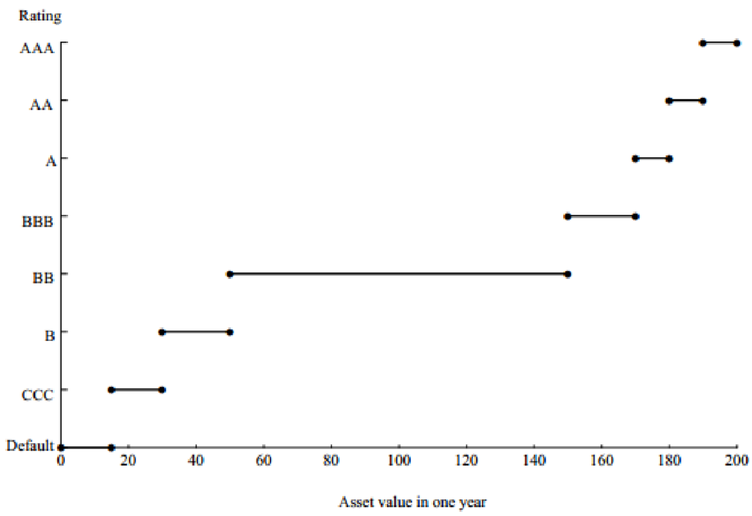
$$C = \begin{bmatrix} \sigma^2(X_1) & \sigma^2(X_1;X_2) & \cdots & \sigma^2(X_1;X_n) \\ \sigma^2(X_2;X_1) & \sigma^2(X_2) & \cdots & \sigma^2(X_2;X_n) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^2(X_m;X_1) & \sigma^2(X_m;X_2) & \cdots & \sigma^2(X_n) \end{bmatrix}. \tag{3.37}$$

Moreover, it is also necessary to build another matrix *W(m+n; n)*, which captures individual bonds and their associated factors, then it is able to calculate the final matrix *A(n; n)*.

Asset value model

Asset value model is based on the proposal that a company's asset value drives its credit rating changes and defaults. Because the value of a company's asset determines the ability to meet its obligations, and if the value of the company's asset is too much low to meet its obligations, the company will default.

Figure 3.6: Credit rating migration driven by BB company asset value



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 86p.

Now suppose there is a series of levels for asset value at the end of the period and suppose a BB-rated company whose assets are worth \$100 million. As presented in *Figure 3.6* above, the asset value of a company in a specific year determines the credit rating of this company. Moreover, the greater the asset value, the higher the credit rating.

Let us parameterize the asset value process to model the change in company's asset value to evaluate its credit rating, namely the percent changes in asset value are normally distributed, the mean is denoted by μ, and the standard deviation is denoted by σ. Besides, given the fact that the value of μ will not influence the final result of the exposition, we can assume $\mu = 0$ to make it easier. Use Z_{Def}, Z_{CCC}, Z_B, etc. to satisfy the situations in which if $R < Z_{Def}$, the company will default; if $Z_{Def} < R < Z_{CCC}$, the company will be re-rated to CCC; if $Z_{CCC} < R < Z_B$, the company will be re-rated to B; and so forth.

Tab 3.11: Transition probabilities and thresholds for a BB-rated company

Rating	Probability from the transition matrix (%)	Cumulative Probability (%)	Threshold
Default	$\Phi(Z_{Def}/\sigma)$= 1.06	1.06	-2.30 σ
CCC	$\Phi(Z_{CCC}/\sigma)-\Phi(Z_{Def}/\sigma)$= 1.00	2.06	-2.04 σ
B	$\Phi(Z_B/\sigma)-\Phi(Z_{CCC}/\sigma)$= 8.84	10.90	-1.23 σ
BB	$\Phi(Z_{BB}/\sigma)-\Phi(Z_B/\sigma)$= 80.53	91.43	1.37 σ
BBB	$\Phi(Z_{BBB}/\sigma)-\Phi(Z_{BB}/\sigma)$= 7.73	99.16	2.39 σ
A	$\Phi(Z_A/\sigma)-\Phi(Z_{BBB}/\sigma)$= 0.67	99.83	2.93 σ
AA	$\Phi(Z_{AA}/\sigma)-\Phi(Z_A/\sigma)$= 0.14	99.97	3.43 σ
AAA	$1-\Phi(Z_{AA}/\sigma)$= 0.03	100.00	

Tab 3.11 presents the transition probabilities of a BB-rated company, and the probability of each rating can be computed as:

$$Pr\{Default\} = Pr\{R < Z_{Def}\} = \Phi(Z_{Def}/\sigma), \tag{3.38}$$

$$Pr\{CCC\} = Pr\{Z_{Def} < R < Z_{CCC}\} = \Phi(Z_{CCC}/\sigma) - \Phi(Z_{Def}/\sigma), \tag{3.39}$$

and so on, where Φ denotes the cumulative distribution for the standard normal distribution.

Take the first row in *Tab 3.11* as an example, the equation can be expressed as:

$$Pr\{Default\} = Pr\{R < Z_{Def}\} = \Phi(Z_{Def}/\sigma) = 1.06\% .$$

Then we can obtain the following equation derived by the equation above:

$$Z_{Def} = \Phi^{-1}(1.06\%) \cdot \sigma = -2.30\sigma ,$$

where $\Phi^{-1}(p)$ denotes the level below which a standard normal distributed random variable falls with probability *p*. And remaining values of threshold can be calculated in the same way.

Similarly, it is easy to obtain the transaction probabilities and asset value thresholds for a single-A rated company as shown in *Tab 3.12*:

Tab 3.12: Transition probabilities and thresholds for an A-rated company

Rating	Probability from the transition matrix (%)	Cumulative Probability (%)	Threshold
Default	$\Phi(Z'_{Def}/\sigma') = 0.06$	0.06	$-3.24\sigma'$
CCC	$\Phi(Z'_{CCC}/\sigma') - \Phi(Z'_{Def}/\sigma') = 0.01$	0.07	$-3.19\sigma'$
B	$\Phi(Z'_{B}/\sigma') - \Phi(Z'_{CCC}/\sigma') = 0.26$	0.33	$-2.72\sigma'$
BB	$\Phi(Z'_{BB}/\sigma') - \Phi(Z'_{B}/\sigma') = 0.74$	1.07	$-2.30\sigma'$
BBB	$\Phi(Z'_{BBB}/\sigma') - \Phi(Z'_{BB}/\sigma') = 5.52$	6.59	$-1.51\sigma'$
A	$\Phi(Z'_{A}/\sigma') - \Phi(Z'_{BBB}/\sigma') = 91.05$	97.64	$1.98\sigma'$
AA	$\Phi(Z'_{AA}/\sigma') - \Phi(Z'_{A}/\sigma') = 2.27$	99.91	$3.12\sigma'$
AAA	$1 - \Phi(Z'_{AA}/\sigma') = 0.09$	100.00	

The evolution of the two credit rating jointly is based on the assumption that the two asset returns are correlated and normally distributed, and there is only the specific correlation ρ between the two asset returns. Therefore, the covariance matrix for the bivariate normal distribution can be computed as:

$$\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma\sigma' \\ \rho\sigma\sigma' & \sigma'^2 \end{pmatrix}, \tag{3.40}$$

Then under the assumption that the correlation ρ between the two asset returns is not equal to zero, the probability that both companies remain in their current credit rating (the asset return for the BB-rated company falls between Z_B and Z_{BB}, while the asset return for the A-rated company falls between Z'_{BBB} and Z'_A) can be computed as:

$$Pr\{Z_B < R < Z_{BB}, Z'_{BBB} < R' < Z'_A\} = \int_{Z_B}^{Z_{BB}} \int_{Z'_{BBB}}^{Z'_A} f(r, r'; \Sigma)(dr')dr, \tag{3.41}$$

where $f(r, r'; \Sigma)$ denotes the density function[9] for the bivariate normal distribution with covariance matrix Σ, and r and r' denote the values that the two asset returns may take on in the specific intervals.

Monte Carlo simulations

Monte Carlo simulations, developed by Stanislaw Ulam and John Von Neumann, are designed to estimate the parameters of a particular probability distribution from the historical data and then the extraction of *N* simulated values for the risk factors.

The Cholesky decomposition, also named Cholesky factorization, is commonly used in the Monte Carlo simulations. In the case of two variables only, A and B, the covariance matrix can be decomposed as:

---

[9] The density function can be computed as: $f(x; y; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}}$.

$$\Sigma=\begin{bmatrix}\sigma_A^2 & \sigma_{A,B}^2\\ \sigma_B^2 & \sigma_B^2\end{bmatrix}=\begin{bmatrix}\sigma_A & 0\\ \dfrac{\sigma_{A,B}^2}{\sigma_A} & \sqrt{\sigma_B^2-\left(\dfrac{\sigma_{A,B}^2}{\sigma_A}\right)^2}\end{bmatrix}\cdot\begin{bmatrix}\sigma_A & \dfrac{\sigma_{A,B}^2}{\sigma_A}\\ 0 & \sqrt{\sigma_B^2-\left(\dfrac{\sigma_{A,B}^2}{\sigma_A}\right)^2}\end{bmatrix}=AA'. \quad (3.42)$$

Similarly, the correlation matrix can be decomposed as:

$$\sum=\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ \rho & \left(1-\rho^2\right)^{1/2}\end{pmatrix}\cdot\begin{pmatrix}1 & \rho\\ 0 & \left(1-\rho^2\right)^{1/2}\end{pmatrix}. \quad (3.43)$$

The following equations are often used to calculate individual elements of the Cholesky decomposition matrix:

$$p_{ii}=\left(\sigma_{ii}-\sum_{k=1}^{i-1}p_{ki}^2\right)^{\frac{1}{2}}, \qquad for\, i=1,2,\ldots,N, \quad (3.44)$$

$$p_{ij}=\left(\sigma_{ij}-\sum_{k=1}^{i-1}p_{ki}\cdot p_{kj}\right)\cdot p_{ii}^{-1}, \qquad for\, i=1,2,\ldots,N, \quad (3.45)$$

$$p_{ij}=0, \qquad for\;\; i>j;i,j=1,2,\ldots,N, \quad (3.46)$$

where p_{ii} and p_{ij} are individual elements of the Cholesky decomposition matrix.

Although the analytical method is accurate, it is only suitable for a small portfolio. Monte Carlo simulation is therefore needed. This method is based on a simulation of a large number of scenarios. Simulation is then performed in three steps:

- Step 1: Generate *m* random values, such as $p_1, p_2, \cdots, p_m$, ranging from 0 to1;
- Step 2: Translate these random values into as many values, such as v_1, v_2, v_3, from a standard normal;
- Step 3: Adjust these values by the equation $x=v\cdot\sigma+\mu$, then it is able to obtain $x_1, x_2, \cdots, x_m$, which reflect the true mean and standard deviation.

Monte Carlo simulations perform risk analysis by estimating models of all possible outcomes obtained by substituting a range of values for any inherently uncertain factors. Then calculate theses outcomes over and over, using a different set of random values each time. Therefore, a single Monte Carlo simulation may involve thousands or tens of thousands of recalculations before it is completed.

3.2.3 Interpretation and application of results

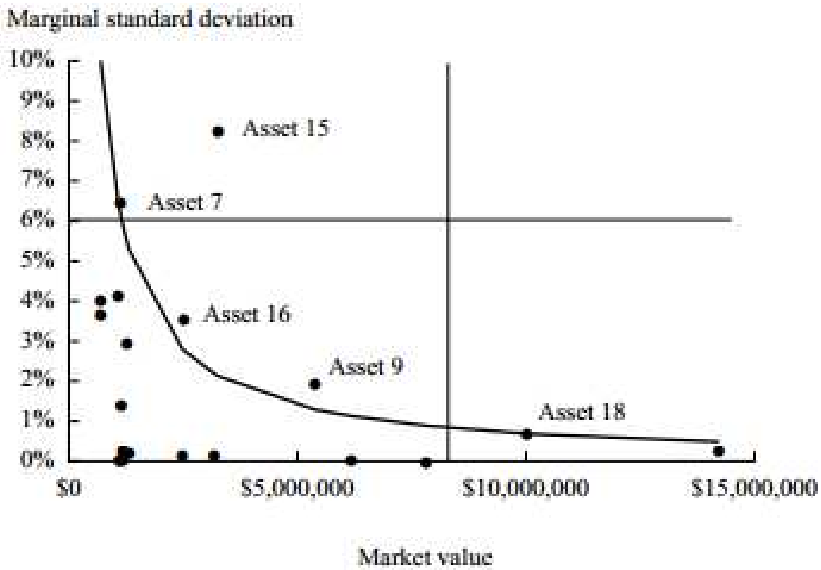
The best way to present the results is using the statistical tools, such as standard deviations and percentiles. Besides, the results can be presented in a graphical way, more specifically, histogram, which is able to estimate the probability distribution of a large volumes of random scenarios in the portfolio by simply connecting the tops of the columns. Another possible graphically way is to conduct a figure, where the x-axis is plotted the credit exposure and the y-axis is the marginal standard deviation expressed as a percentage. The product of these two quantities is the marginal value of the absolute risk. It is also appropriate to insert the curve showing the same level of risk, which is named iso – risk line, to identify the bonds that contribute most to the overall risk of the portfolio because these bonds are usually above the line. After determining the amount of the portfolio risk and marginal risk of individual instruments, it is possible to reduce the risk associated with the portfolio. One effective way to accomplish the reduction is to define the credit risk limits.

Credit risk limits

There are three possible risk limits, including:

- Based on the percentage risk (the horizontal line in *Figure 3.7*). This limit can set a restriction to the exposures that are more correlated to the portfolio.
- Based on the exposure size (the vertical line in *Figure 3.7*). This limit can set a restriction to the portfolio to have no exposures above a given size regardless of the quality of credit.
- Based on the absolute risk (the curve in *Figure 3.7*). This limit can cap the total risk of the portfolio at a given amount above the current risk.

Figure 3.7: Possible risk limits for an example portfolio



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 135p.

Each portfolio manager is not able to determine the credit limits alone, and these credit limits are not exceeded the acceptance of the bond in every situation. This is because of the existence of other forms of collateral, which is, for example, hedging by using the credit derivatives. Therefore, it is necessary to distinguish:

- soft limits – especially informative and may require more detailed exploration or security,
- hard limits – able to prevent the entry of a particular borrower, industry, region, or financial instrument in the portfolio.

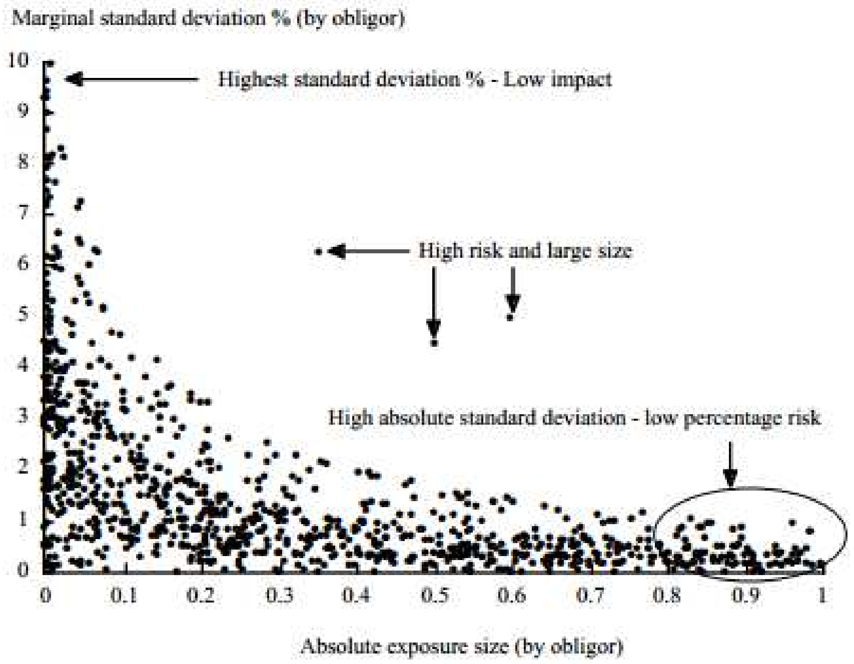
Prioritizing risk reduction actions

Given the fact that there exist many actions that may be taken towards addressing risk, these actions need prioritizing, especially in a certain portfolio with a large number of exposures. Originally, there are two features of risk that are worth reducing, namely absolute exposure size and statistical risk level.

Three essential measures include:

- Obligors with the largest absolute exposure sizes (the lower right corner in *Figure 3.8*) would have the greater impact on the portfolio compared with other obligors.
- Obligors with the highest marginal standard deviations (the upper left corner in *Figure 3.8*) would have higher probability to contribute to portfolio losses.
- Obligors with the large absolute exposure sizes and high marginal standard deviations (points towards the upper right corner in *Figure 3.8*) would be the largest contributors to portfolio risk.

Figure 3.8: Risk versus size of exposure within a typical credit portfolio



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 134p.

The third measure is usually set as the highest priority to address the obligors with both relatively high risk and relatively large size. However, the large exposure sizes are allowed only if they have low risks, while the high percentage risks are allowed only if they have small sizes. Considering the credit quality could change and a large exposure could have the credit rating downgraded, the portfolio, therefore, is possible to have both a large exposure size and a high percentage risk level.

3.3 Regulation of capital requirements

Bank regulation has been an evolutionary process since 1988. The role of capital is a central element of regulation for banks and financial institutions, not only because the cost of capital is the important driver behind the calculations of return on capital, but also because meeting the return on capital targets is a prime objective of banking operations. Banks must meet minimum capital requirements under the standard Bank for International Settlements (BIS) before chartered.

3.3.1 Basel I

At the end of 1974, in order to negotiate a common approach of measuring capital adequacy, the Basel Committee on Banking Supervision was created and charged by the largest ten industrialized countries in the world, including Belgium, Canada, France, Italy, Germany, Japan, Netherlands, Sweden, the United Kingdom, and the United States, plus Luxembourg and Switzerland. In July 1988, the first document of this meeting was formally approved and entitled "International Convergence of Capital Measurement and Capital Standards", which was referred to as 1988 Capital Accord (the Accord). This Accord was implement by the end of 1992 and is known as Basel I nowadays.

According to Basel I, the capital has two tiers:

- Tier 1 (core) = common stock and surplus + noncumulative perpetual preferred stock + minority interest in the equity accounts of consolidated subsidiaries + selected identifiable intangible assets - goodwill and other intangible assets.
- Tier 2 (supplemental) = the allowance for loan and lease loans + subordinated debt capital instruments + mandatory convertible debt + intermediate-term preferred stock + cumulative perpetual preferred stock with unpaid dividends + other long-term hybrid capital instruments.

Once we know a certain bank's Tier 1 capital, Tier 2 capital, and total risk-weighted assets for credit risk, the level of capital requirement can be given by:

$$Tier\ 1\ ratio = \frac{Tier\ 1\ capital}{RWA} \geq 4\%, \tag{3.47}$$

$$Capital\ adequacy\ ratio = \frac{Tier\ 1\ capital + Tier\ 2\ capital}{RWA} \geq 8\%, \tag{3.48}$$

Credit risk exposures can be divided into three groups, including those arising from on-balance sheet assets (excluding derivatives), those arising from off-balance sheet items (excluding derivatives), and those arising from over-the-counter derivatives. The on-balance sheet assets, referred to as the capital adequacy risk-weighted assets, can be summarized as follows:

- No risk: 0% (e.g., cash, cash equivalents, or claims on OECD[10] governments such as Treasury bonds),
- Low risk: 20% (e.g., claims on OECD banks and OECD public sector entities such as short-term securities),
- Moderate risk: 50% (e.g., uninsured residential mortgage loans),
- Standard risk: 100% (e.g., all other clams such as commercial loans).

Then the risk-weighted assets for *N* items can be computed by:

$$RWA = \sum_{i=1}^{N} w_i \cdot EAD_i, \tag{3.49}$$

where w_i is the risk weight of the *i*th item and EAD_i is the exposure at default.

Basel I only reflected credit risk from 1988 to 1996, then the 1996 Amendment introduced market risk. Therefore, the capital adequacy ratio (CAR) can be given by:

$$Capital\ adequacy\ ratio = \frac{Tier\ 1\ capital + Tier\ 2\ capital}{RWA + \left(12.5 \cdot CR_m\right)} \geq 8\%, \tag{3.50}$$

where CR_m is capital requirements for market risk.

[10] OECD = Organization of Economic Co-operation and Development.

3.3.2 Basel II

Under the 1988 Basel Accord, all loans by a bank to a corporation have a risk weight of 100% and the amount of the required capital is the same. Specifically, a loan to an AAA-rated corporation and one to a B-rated corporation are treated in the same way. In response to the significant weaknesses of the 1988 Basel Accord, the Basel Committee proposed a new capital adequacy framework, known as Basel II, on 3 June 1999. The consultative document was published in April 2003, and the agreement was reached in May 2004. Finally, Basel II has been valid since January 2007.

Basel II is split into three so-called pillars as follows:

Basel II

Pillar 1 **Minimum capital requirements**	**Pillar 2** **Supervisory review**	**Pillar 3** **Market discipline**
· Credit risk · Market risk · Operational risk	· Assessment of risks and capital adequacy of the individual banks · Constant contact with banks	· Increasing disclosure of capital requirements as well methods of risk assessment

Pillar 1: Minimum capital requirements

Pillar 1 seeks to set capital requirements as a function of the credit, market, and operational risk exposures of the bank. Credit risk is associated with a new measurement, market risk stays unchanged, and operational risk is a new risk to be considered. Therefore, the total risk-weighted assets RWA_T for the bank is:

$$RWA_T = RWA_c + 12.5 \cdot \left(CR_m + CR_o \right), \tag{3.51}$$

where RWA_c is risk-weighted assets for credit risk, CR_m is market risk capital requirements, and CR_o is operational risk capital requirements.

In addition, the regulatory capital (RC) and Basel II minimum capital requirement can be expressed as:

$$RC = Tier\ 1\ capital + Tier\ 2\ capital + Tier\ 3\ capital - deductions\ , \tag{3.52}$$

$$CAR = \frac{RC}{RWA_T} = \frac{RC}{RWA_c + 12.5 \cdot (CR_m + CR_o)} \geq 8\%\ . \tag{3.53}$$

Note that Tier 3 capital includes a wider variety of subordinated debt and can only be used to support market risk; while deductions includes goodwill, a subjective assessment, shares that are held by one bank in another bank, and investments in subsidiaries engaged in banking activities. For credit risk, the eligible Tier 1 and Tier 2 capital must be greater than $8\% \cdot RWA_c$ (or CR_c instead); while for market risk, the eligible Tier 1, Tier 2, and Tier 3 capital must be greater than CR_m.

Basel II defines several possible approaches to calculate capitals from basic approaches to more sophisticated approaches as presented in *Tab 3.13*. For credit risk, there are two major approaches to state the capital requirements, including the standardized approach and the internal ratings-based (IRB) approach. There is a foundation IRB approach and an advanced IRB approach within the IRB approach, and the advanced one gives banks more space to set parameters themselves.

Tab 3.13: Methods for calculating capital according to Basel II

	Credit risk	**Market risk**	**Operational risk**
Approaches	· Standardized Approach · Foundation Internal Ratings-Based (IRB) Approach · Advanced IRB Approach	· Standardized Approach · Internal Models Approach	· Basic Indicator Approach · Standardized Approach · Advanced Measurement Approach
Result	Risk-weighted asset value for credit risk	Market risk capital charge	Operational risk capital charge

Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 203p.

Standardized approach

In the standardized approach, risk-weighted assets are split according to the formal credit ratings associated with a set matrix as *Tab 3.14* shows.

Tab 3.14: Capital requirement risk weights under Basel II

	Government	Public sector	Banks	Corporations
AAA to AA-	0%	20%	20%	20%
A+ to A-	20%	50%	50%	50%
BBB+ to BBB-	50%	100%	100%	100%
BB+ to B-	100%	100%	100%	100%
B+ to B-	100%	150%	150%	150%
Below B-	100%	150%	150%	150%
Unrated	100%	100%	100%	100%

Source: BIS.

Under Basel II, the OECD status the risk weight for a government exposure ranges from 0% to 150% and the risk weight for an exposure to public sector, banks, or corporations ranges from 20% to 150%.

Under Basel I, OECD banks were assumed implicitly to be lesser credit risks than corporations. In other words, an OECD bank was associated with a risk weight of 20% while a corporation was associated with a risk weight of 100%. However, as shown in *Tab 3.14* above, banks and corporations are treated more equitably. If a bank, for example, is rated between AAA and AA, the risk weight assigned to this bank will be 20%. Then according to equation (3.49), it is possible to obtain the risk-weighted assets for credit risk.

Internal ratings-based (IRB) approach

In the internal ratings-based approach, risk-weighted assets are categorized according to the internal risk assessment especially for banks that must have their own internal systems to categorize loans in PD bands as shown in *Tab 3.15*. The IRB approach can be categorized into the foundation IRB approach and the advanced IRB approach.

Tab 3.15: Capital requirements under specified PD bands (%)

	PD band	Basel I	Standard approach	IRB foundation approach
AAA	0.03	8.0	1.6	1.13
AA	0.03	8.0	1.6	1.13
A	0.03	8.0	4.0	1.13
BBB	0.20	8.0	8.0	3.61
BB	1.40	8.0	8.0	12.35
B	6.60	8.0	12.0	30.96
CCC	15.00	8.0	12.0	47.04

Source: BIS.

Under the foundation IRB approach, senior unsecured claims on sovereigns and banks are assigned a 45% LGD, while subordinated unsecured debts are assigned a 75% LGD. Maturity (M) is usually assigned 2.5. On the other hand, under the advanced IRB approach, banks will use their own values for PDs, LGDs, EADs, and Ms. In addition, the formulas under both the foundation and the advanced IRB approaches for calculating the risk-weights, capital requirements, loss given default, and risk-weighted assets can be given by:

$$RWA = CR \cdot 12.5 \cdot EAD, \tag{3.54}$$

$$\begin{aligned} &Capital\ Requirement\ (CR) = \\ &\left[LGD \cdot N\left((1-R)^{0.5} \cdot G(PD) + \left[\frac{R}{(1-R)}\right]^{0.5} \cdot G(0.999) \right) - (PD \cdot LGD) \right] \\ &\cdot \left(\frac{1}{1-1.5 \cdot b}\right) \cdot \left[1 + (M-2.5) \cdot b\right] \end{aligned} \tag{3.55}$$

$$LGD^* = max\left[0; LGD \cdot \left(\frac{E^*}{E}\right)\right], \tag{3.56}$$

$$\begin{matrix} Correlation \\ (R) \end{matrix} = 0.12 \cdot \frac{\left[1-exp(-50 \cdot PD)\right]}{\left[1-exp(-50)\right]} + 0.24 \cdot \left[1 - \frac{\left[1-exp(-50 \cdot PD)\right]}{\left[1-exp(-50)\right]}\right], \tag{3.57}$$

$$Maturity\ adjustment\,(b) = \left(0.11825 - 0.05478 \cdot ln(PD)\right)^2, \tag{3.58}$$

where E is the present value of the exposure, E^* is the value after hedging, $N(\ )$ is the cumulative distribution function for a standard normal variable with $N(0,1)$ and $G(z)$ is the inverse cumulative distribution function for a standard normal variable, which means $N(x) = z$.

Note that equation (3.47) is only suitable for retail banking. If there is corporate banking with sales more than 5 million euro, the calculation of the correlation R' should be slightly different as follows:

$$R' = R + 0.04 \cdot \left[1 - \left(\frac{S-5}{45}\right)\right], \tag{3.59}$$

where S denotes sales.

Pillar 2: Supervisory review

Pillar 2 describes the supervisory review process, designed to ensure banks meet the minimum capital requirements. Supervisors are responsible for evaluating how well banks are coping with their internal capital adequacy assessments. Moreover, supervisors expect banks to operate above the minimum regulatory capital and hold more capital than the minimum requirement. Lastly, supervisors should intervene to prevent the capital drops below the minimum requirement at an early stage.

Pillar 3: Market discipline

Pillar 3 defines the disclosure requirements that allow the banks' risk assessment procedures and capital adequacy to be readily assessed. The disclosures include: (i) the economic and financial results, (ii) the financial structure, (iii) risk management techniques, (iv) risk exposures to different risk types, (v) capital adequacy, and (vi) management and corporate governance.

3.3.3 Basel III

The Basel Committee realized the necessity to overhaul Basel II after the 2007-2009 credit crisis. In December 2009, Basel III was first published because the Basel Committee wanted to tighten the definition of capital and address liquidity risk. Then the final version of Basel III was published in December 2010 by the Basel Committee for Banking Supervision (BCBS), which consists of the regulators and central bankers of 27 countries. *Tab 3.16* below summarizes Basel III phase-in arrangements, which is proposed by BIS.

Tab 3.16: Basel III phase-in arrangements (all dates are as of 1 January)

	Phases	2013	2014	2015	2016	2017	2018	2019
Capital	Leverage ratio		a[11]				b[12]	
Capital	Minimum common equity capital ratio	3.5%	4.0%	4.5%				**4.5%**
Capital	Capital conservation buffer				0.625%	1.25%	1.875%	**2.50%**
Capital	Minimum common equity plus capital conservation buffer	3.5%	4.0%	4.5%	5.125%	5.75%	6.375%	**7.0%**
Capital	Phase-in of deductions from CET1*		20%	40%	60%	80%	100%	**100%**
Capital	Minimum Tier 1 capital	4.5%	5.5%	6.0%				**6.0%**
Capital	Minimum total capital		8.0%					**8.0%**
Capital	Minimum total capital plus conservation buffer		8.0%		8.625%	9.25%	9.875%	**10.5%**
Capital	Capital instruments that no longer qualify as non-core Tier 1 capital or Tier 2 capital		Phased out over 10 year horizon beginning 2013					
Liquidity	Liquidity coverage ratio - minimum requirement			60%	70%	80%	90%	**100%**
Liquidity	Net stable funding ratio						c[13]	

**Including amounts exceeding the limit for deferred tax assets (DTAs), mortgage servicing rights (MSRs) and financials.*

- - - transition periods

Source: BIS.

[11] Parallel run 1 Jan 2013 - 1 Jan 2017, Disclosure starts 1 Jan 2015.

[12] Migration to Pillar 1.

[13] Introduce minimum standard.

In summary, the main six provisions of Basel III includes:

- the minimum level of core Tier 1 capital ratio must be 4.5% all the time;
- a capital conservation buffer of 2.5% is required to protect against economic and financial stress, which means the minimum level of total capital must be 13% all the time;
- the minimum level of total capital must be 8% all the time;
- a Tier 1 leverage ratio of 3%;
- two liquidity ratios, liquidity coverage ratio (LCR) and net stable funding ratio (NSFR), are added to determine banks can survive liquidity pressures;
- a risk-weighting of 1%-3% on the market-to-market and collateral exposures to the counterparty credit risk.

4 Determination of credit risk by selected models

To this point, we have detailed the basic descriptions of financial risks, credit risk management, and some useful models. In this chapter, we practice these theories for the sake of a better understanding. Capital requirements for the expected loss caused by credit risk are calculated both under Basel agreements and by CreditMetrics™ model respectively.

Firstly, the input data and the portfolio will be described. Ten different companies are selected. Then, the calculations of the size of the regulatory capital will be conducted under Basel agreements, using the standard approach (SA) and the foundation internal ratings-based approach (FIRB). Furthermore, the calculations of economic capital according to CreditMetrics™ model will be presented. Finally, the results will be concluded and compared. Time horizon to calculate the capital requirements that is aimed to cover the unexpected losses is one year and starts at the beginning of January 1st, 2016.

4.1 Input data

The comparison of methods for calculating the capital requirements is performed in a portfolio that consists of ten different debt assets traded on the Frankfurt Stock Exchange (FSE) with a total nominal value of 10 million euro. Each bond, therefore, is represented equally in a nominal value of 1 million euro in order to avoid bias caused by high nominal values of some bonds. All basic information about the issued bonds can be accessed on the official website of the Frankfurt Stock Exchange.

The important information mainly includes the nominal value, the size of the coupon, the maturity date, the market price, ratings, and seniority. All bonds have a seniority Senior Unsecured, because they are bonds of renowned companies, which are not required to cover mortgage bonds. The basic information about these bonds are presented in *Tab 4.1* below.

Tab 4.1: Basic information about individual bonds

Name	Rating	Coupon	Nominal value	Maturity	Market price	pcs.
Deutsche Post	A-	2.75%	1,000 €	10/2023	113.67%	1,000
E.ON	A-	5.80%	1,000 €	4/2018	106.69%	1,000
Metro	BBB-	1.50%	1,000 €	3/2025	100.01%	1,000
Volkswagen	A+	2.37%	100,000 €	9/2022	105.80%	10
NIKE	AA-	2.25%	2,000 €	5/2023	99.98%	500
Commerzbank	BBB+	0.08%	1,000 €	6/2023	97.34%	1,000
Bayer	A-	1.87%	1,000 €	1/2021	107.25%	1,000
Nestle Holdings	AA	4.25%	2,000 €	3/2020	104.26%	500
Danone	BBB+	3.00%	200,000 €	6/2022	101.54%	5
Oracle	A+	2.80%	2,000 €	7/2021	102.81%	500

Source: Frankfurt Stock Exchange (FSE).

As shown in *Tab 4.1*, all bonds are denominated in euros (€) and the nominal values are between 1,000 € and 200,000 €. The ratings are provided by the rating agency Standard & Poor's (S & P). Issuers of these bonds are all large companies with high credit ratings. The highest rating among them is AA, while the lowest rating is BBB-.

It is necessary to know the probability of default of individual bonds with different credit ratings when determining the capital requirements. These probabilities are obtained on the basis of the transition matrix for European companies issued by Standard & Poor's as presented in *Tab 4.2*.

Tab 4.2: The probability of default for different ratings

Rating	PD	Rating	PD
AAA	0.0007%	**BBB-**	0.2747%
AA+	0.0022%	**BB+**	0.7117%
AA	0.0024%	**BB**	1.2581%
AA-	0.0044%	**BB-**	4.1917%
A+	0.0142%	**B+**	8.8480%
A	0.1075%	**B**	24.4180%
A-	0.2020%	**B-**	48.6187%
BBB+	0.2045%	**CCC**	
BBB	0.2730%		

Source: Standard & Poor's.

Given the fact that every bond is Senior Unsecured, thus the recovery rate is 51.13%. According to the equation (2.1), the loss given default is 48.87%. The rate of return is obtained from Carty & Lieberman and can be found in *Tab 2.4* in the previous chapter.

4.2 Calculating credit risk under Basel I, II, and III

Now it is possible to determine the size of capital requirements to cover the unexpected losses from credit risks through methods under the Basel agreements. The calculation methodology is described in subchapter 3.3. Each bond in the portfolio is represented in the same weight, which corresponds to a nominal value of 1 million euro.

Under Basel I

The size of capital requirements is determined under Basel I at first. The individual bonds are assigned risk weights, and then according to the equation (3.49), it is simple to obtain the size of risk-weighted assets. The capital requirements can be obtained once the risk-weighted assets are known. The results are presented in *Tab 4.3* as below.

Tab 4.3: Regulatory capital requirements under Basel I

Basel I	Rating	Nominal value	w	RWA	CR
Deutsche Post	A-	1,000,000 €	100%	1,000,000 €	80,000 €
E.ON	A-	1,000,000 €	100%	1,000,000 €	80,000 €
Metro	BBB-	1,000,000 €	100%	1,000,000 €	80,000 €
Volkswagen	A+	1,000,000 €	100%	1,000,000 €	80,000 €
NIKE	AA-	1,000,000 €	100%	1,000,000 €	80,000 €
Commerzbank	BBB+	1,000,000 €	20%	200,000 €	16,000 €
Bayer	A-	1,000,000 €	100%	1,000,000 €	80,000 €
Nestle Holdings	AA	1,000,000 €	100%	1,000,000 €	80,000 €
Danone	BBB+	1,000,000 €	100%	1,000,000 €	80,000 €
Oracle	A+	1,000,000 €	100%	1,000,000 €	80,000 €
Total	-	-	-	**9,200,000 €**	**736,000 €**

The results above indicate a fundamental shortage of Basel I, namely there is no consideration on the borrower's credit eligibility. Except Commerzbank, almost all bonds have a risk weight of 100%. Therefore, the value of the total risk-weighted assets is 9.2 million euro and the value of the regulatory capital requirements is 736 thousand euro.

Under Basel II

Secondly, the size of capital requirements is determined under Basel II, including both the standard approach and the foundation internal ratings-based approach. The procedure for the calculation based on the standard approach is the same as the procedure in the last case and the accordingly results are shown in *Tab 4.4*.

Tab 4.4: Regulatory capital requirements under Basel II - SA

Basel II - SA	Rating	Nominal value	w	RWA	CR
Deutsche Post	A-	1,000,000 €	50%	500,000 €	40,000 €
E.ON	A-	1,000,000 €	50%	500,000 €	40,000 €
Metro	BBB-	1,000,000 €	100%	1,000,000 €	80,000 €
Volkswagen	A+	1,000,000 €	50%	500,000 €	40,000 €
NIKE	AA-	1,000,000 €	20%	200,000 €	16,000 €
Commerzbank	BBB+	1,000,000 €	100%	1,000,000 €	80,000 €
Bayer	A-	1,000,000 €	50%	500,000 €	40,000 €
Nestle Holdings	AA	1,000,000 €	20%	200,000 €	16,000 €
Danone	BBB+	1,000,000 €	100%	1,000,000 €	80,000 €
Oracle	A+	1,000,000 €	50%	500,000 €	40,000 €
Total	-	-	-	**5,900,000 €**	**472,000 €**

The Basel II has improves the shortage of Basel I and the size of risky assets is based on the borrower's credit eligibility. The values of the risk-weighted assets for most bonds have changes because of the changes in the risk weights, which contributes a decline in the overall value of risk-weighted assets by 3.3 million euro. The value of the regulatory capital requirements, from 736 thousand euro to 472 thousand euro, then decreases by almost 35.87%.

Furthermore, the size of capital requirements can be determined under Basel II by the foundation internal ratings-based approach. According to a series of equations from (3.55) to (3.58), it is able to obtain the values of both the capital requirements and the total risk-weighted assets. The results are presented in *Tab 4.5*.

Tab 4.5: Regulatory capital requirements under Basel II - FIRB

Basel II - FIRB	Rating	RWA	CR
Deutsche Post	A-	479,283 €	38,343 €
E.ON	A-	479,283 €	38,343 €
Metro	BBB-	564,392 €	45,151 €
Volkswagen	A+	100,317 €	8,025 €
NIKE	AA-	52,235 €	4,179 €
Commerzbank	BBB+	482,497 €	38,600 €
Bayer	A-	479,283 €	38,343 €
Nestle Holdings	AA	39,314 €	3,145 €
Danone	BBB+	482,497 €	38,600 €
Oracle	A+	100,317 €	8,025 €
Total	-	**3,259,417 €**	**260,753 €**

The main aim of introducing the foundation internal ratings-based approach is to encourage banks to use more sophisticated methods to calculate regulatory capital requirements to better reflect the risks incurred. The motivation to use FIRB approach involves lower capital requirements when compared with the SA in the previous case. Specifically speaking, the value of the capital requirements of Nestle Holdings in the case of using FIRB approach decreases by nearly 80.34% from 16,000 € to 3,145 €. Moreover, in the view of the overall portfolio, the absolute change of the value of capital requirements is 211,247 € and the change expressed in a percentile is almost 44.76%.

Under Basel III

Thirdly, the size of capital requirements is determined under Basel III, including both the standard approach and the foundation internal ratings-based approach as Basel II. The procedure for the calculation based on the standard approach is the same as the procedure under Basel II, but the minimum level of the capital adequacy ratio becomes 13%, 10.5% plus a countercyclical buffer of 2.5%, under

Basel III. Thus, it is necessary to calculate the minimum level of capital requirements without the countercyclical buffer. The results are presented in *Tab 4.6*.

Tab 4.6: Regulatory capital requirements under Basel III - SA

Basel III - SA	Rating	Nominal value	w	RWA	CR
Deutsche Post	A-	1,000,000 €	50%	500,000 €	52,500 €
E.ON	A-	1,000,000 €	50%	500,000 €	52,500 €
Metro	BBB-	1,000,000 €	100%	1,000,000 €	105,000 €
Volkswagen	A+	1,000,000 €	50%	500,000 €	52,500 €
NIKE	AA-	1,000,000 €	20%	200,000 €	21,000 €
Commerzbank	BBB+	1,000,000 €	100%	1,000,000 €	105,000 €
Bayer	A-	1,000,000 €	50%	500,000 €	52,500 €
Nestle Holdings	AA	1,000,000 €	20%	200,000 €	21,000 €
Danone	BBB+	1,000,000 €	100%	1,000,000 €	105,000 €
Oracle	A+	1,000,000 €	50%	500,000 €	52,500 €
Total	-	-	-	**5,900,000 €**	**619,500 €**

From the results shown in *Tab 4.6* above, it is obvious that the value of the total risk-weighted assets under Basel III – SA is the same as that under Basel II – SA (shown in *Tab 4.4*). The value of the capital requirements, however, increases by 31.25% from 472,000 € to 619,500 € due to the increase of the minimum requirement for capital adequacy by 2.5%. On the other hand, when compared with the results under Basel I, the values of the total risk-weighted assets and the capital requirements decrease by 35.87% and 15.83% respectively.

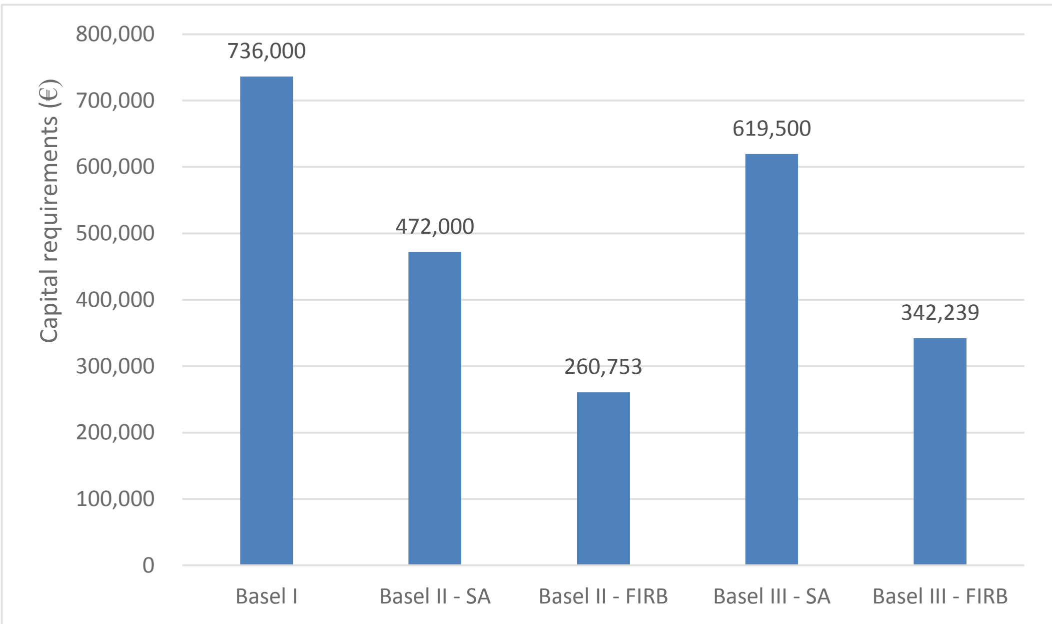
Then the size of capital requirements can be determined under Basel III by the foundation internal ratings-based approach. The calculation is almost the same as under Basel II – FIRB except the minimum level of the capital adequacy ratio increases from 8% to 10.5%. The results are presented in *Tab 4.7*. When comparing the foundation internal ratings-based approach with the standard approach under Basel III, the absolute changes of the values of both the total risk-weighted assets and the capital requirements are even higher than the absolute changes under Basel II. Specifically, the regulatory capital requirements declines by 277,261 € under Basel III, while that declines by 211,247 € under Basel II.

Tab 4.7: Regulatory capital requirements under Basel III - FIRB

Basel III - FIRB	Rating	RWA	CR
Deutsche Post	A-	629,058 €	50,325 €
E.ON	A-	629,058 €	50,325 €
Metro	BBB-	740,765 €	59,261 €
Volkswagen	A+	131,666 €	10,533 €
NIKE	AA-	68,558 €	5,485 €
Commerzbank	BBB+	633,277 €	50,662 €
Bayer	A-	629,058 €	50,325 €
Nestle Holdings	AA	51,600 €	4,128 €
Danone	BBB+	633,277 €	50,662 €
Oracle	A+	131,666 €	10,533 €
Total	-	**4,277,985 €**	**342,239 €**

In summary, the following *Figure 4.1* concludes the regulatory capital requirements under different agreements and approaches. The results indicate that it is definitely worthwhile for banks to use more sophisticated methods to calculate the capital requirements.

Figure 4.1: Regulatory capital requirements under Basel I, II, and III



4.3 Calculating credit risk by CreditMetrics™

In this subchapter, CreditMetrics™ model is applied into practice in details. The first step is calculating the yields derived from the combination of both the covariance matrix and the correlation matrix. Then determine the values of bonds for each grade and use the forward yield curves derived from the transition matrix. The transition matrix is supposed to derive the bounds of transitions among rating categories. Next, conduct the Monte Carlo simulation. It will generate 25,000 random yields for each bond. The sum of these random yields and the Cholesky decomposition matrix will be obtained by the values of correlated returns. Each individual yield is based on the transition between assigned rating categories and the obtained rating of each bond will be assigned an appropriate value. Adding the values of individual bonds makes it possible to obtain the value of the overall portfolio. Lastly, the final results are calculated using the risk characteristics.

4.3.1 Estimation of the correlation among bonds issuers

The estimation of the correlation among individual issuers is based on the market prices of shares of each issuer. The selected time horizon of the value of the shares on each trading day is the period from March 26th, 2015 to March 16th, 2016, which is presented in Annex 2.

Tab 4.8: Correlations among individual issuers

	DP	E.ON	Metro	VW	NIKE	Comm.	Bayer	Nestle	Danone	Oracle
DP	1.00	0.51	0.60	0.44	0.35	0.53	0.72	0.53	0.55	0.17
E.ON	0.51	1.00	0.53	0.46	0.20	0.51	0.55	0.42	0.44	0.07
Metro	0.60	0.53	1.00	0.46	0.26	0.50	0.64	0.57	0.64	0.12
VW	0.44	0.46	0.46	1.00	0.10	0.42	0.42	0.36	0.30	0.14
NIKE	0.35	0.20	0.26	0.10	1.00	0.25	0.35	0.37	0.29	0.10
Comm.	0.53	0.51	0.50	0.42	0.25	1.00	0.59	0.42	0.36	-0.04
Bayer	0.72	0.55	0.64	0.42	0.35	0.59	1.00	0.60	0.61	0.16
Nestle	0.53	0.42	0.57	0.36	0.37	0.42	0.60	1.00	0.67	0.20
Danone	0.55	0.44	0.64	0.30	0.29	0.36	0.61	0.67	1.00	0.21
Oracle	0.17	0.07	0.12	0.14	0.10	-0.04	0.16	0.20	0.21	1.00

Firstly, determine the covariance matrix and the correlation matrix from the yields of selected individual shares. Both these two matrixes can be obtained by the analytical tools of MS Excel – Data/Data Analysis. The results can be found in Annex 3. The correlations among individual issuers derived from the covariance matrix and the correlation matrix is shown in *Tab 4.8* above. The high correlations between two issuers indicate that these two companies operate in the same or highly related industry. For example, the value of the correlation between Nestle and Danone is 0.67 as presented in *Tab 4.8* mainly due to the fact that both Nestle and Danone are world leading food companies originated from Europe.

4.3.2 Calculation of the values of bonds

Secondly, it is necessary to determine the present values of the selected bonds. In order to obtain the present values, the yield curve needs deriving, which is described in subchapter 3.2.2. So it is important to create a multiannual transition matrix and know the risk-free rate, probability of default and recovery rate. The multiannual transition matrix is obtained from the annual transition matrix, which can be found in Annex 1, by squaring the desired exponent. These multiannual transition matrixes are presented in Annex 4. The individual probability of default are always the last column of each multiannual transition matrix. The recovery rate is selected according to Carty and Lieberman at 51.13%. The values of the risk-free rate from 2016 to 2025 are derived from the interest rate swaps (IRS), which can be found on the official website of the Deutsche Bundesbank. According to the equation (3.23), it is able to calculate the forward rates. Both the spot rates and the forward rates from 2016 to 2025 are presented in *Tab 4.9*.

Tab 4.9: Spot rate (IRS) and forward rates (f_n^F) from 2016 to 2025 (%)

Year	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
IRS	-0.16	-0.18	-0.12	-0.03	0.08	0.20	0.32	0.45	0.56	0.68
f_n^F	-0.16	-0.19	-0.01	0.23	0.51	0.82	1.02	1.38	1.49	1.69

Source: Deutsche Bundesbank

With the knowledge of all the necessary input data, it is possible to use the equation (3.27) to compute the yield curve for bonds, according to all maturities and credit ratings. The result of the forward yield curve is presented in Annex 5.

Then the present values of bonds can be determined according to individual rating categories by applying the equation (3.19). These values are listed in *Tab 4.10*.

Tab 4.10: Present values of bonds according to the rating categories

Bond	DP	E.ON	Metro	VW	NIKE	Comm.	Bayer	Nestle	Danone	Oracle
AAA	1,109	1,175	988	109,410	2,140	902	1,063	2,372	227,388	2,236
AA+	1,109	1,175	987	109,386	2,140	902	1,063	2,372	227,341	2,236
AA	1,109	1,175	987	109,385	2,140	902	1,063	2,372	227,338	2,236
AA-	1,108	1,175	987	109,359	2,139	901	1,063	2,371	227,285	2,236
A+	1,108	1,174	986	109,304	2,138	901	1,063	2,371	227,172	2,235
A	1,105	1,173	983	109,069	2,133	899	1,060	2,366	226,689	2,230
A-	1,105	1,173	983	109,090	2,133	899	1,061	2,367	226,735	2,231
BBB+	1,102	1,172	980	108,798	2,127	896	1,058	2,363	226,136	2,226
BBB	1,099	1,171	977	108,546	2,121	893	1,056	2,359	225,620	2,221
BBB-	1,093	1,166	971	107,972	2,109	888	1,051	2,348	224,438	2,210
BB+	1,093	1,169	970	107,992	2,109	888	1,051	2,350	224,488	2,211
BB	1,081	1,159	959	106,817	2,085	877	1,040	2,328	222,066	2,189
BB-	1,049	1,141	927	103,788	2,023	850	1,013	2,274	215,840	2,132
B+	1,028	1,124	909	101,733	1,983	832	993	2,232	211,601	2,090
B	983	1,080	870	97,241	1,896	795	949	2,136	202,297	1,998
B-	842	948	743	83,256	1,622	677	813	1,841	173,350	1,714
CCC	679	746	602	67,099	1,309	549	654	1,472	139,619	1,377
D	511	511	511	51,130	1,023	511	511	1,023	102,260	1,023

The colorful cells denotes the default values if bonds with the assigned credit ratings. The values of bonds in the case of default can be determined by multiplying the recovery rate and the size of the exposure. Moreover, *Tab 4.10* also indicates that the present value of a certain bond is lower with a lower credit rating.

4.3.3 Simulation of the value of the portfolio

Thirdly, within the Monte Carlo simulation, it is necessary to generate a series of random yields. This can be realized by using the function of MS Excel – Data/Data Analysis/Random Number Generator. The standard normal distribution N (0, 1) is applied to generate these 25,000 random yields for each bond. These scenarios can be found in Annex 6.

Since the individual issuers are independent, it is necessary to take these dependencies into account when simulating the yields, which can be achieved by using an upper triangular Cholesky decomposition matrix as shown in *Tab 4.11*.

Tab 4.11: Cholesky decomposition matrix

	DP	E.ON	Metro	VW	NIKE	Comm.	Bayer	Nestle	Danone	Oracle
DP	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
E.ON	0.5145	0.8575	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Metro	0.6046	0.2519	0.7557	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
VW	0.4417	0.2689	0.1598	0.8409	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
NIKE	0.3514	0.0248	0.0602	-0.0893	0.9297	0.0000	0.0000	0.0000	0.0000	0.0000
Comm.	0.5344	0.2768	0.1405	0.1053	0.0589	0.7769	0.0000	0.0000	0.0000	0.0000
Bayer	0.7231	0.2042	0.2016	0.0172	0.0876	0.1466	0.6044	0.0000	0.0000	0.0000
Nestle	0.5275	0.1683	0.2745	0.0442	0.1839	0.0465	0.1791	0.7403	0.0000	0.0000
Danone	0.5457	0.1856	0.3504	-0.0612	0.0718	-0.0391	0.1698	0.2934	0.6478	0.0000
Oracle	0.1712	-0.0179	0.0319	0.0799	0.0437	-0.1838	0.0922	0.1156	0.0822	0.9480

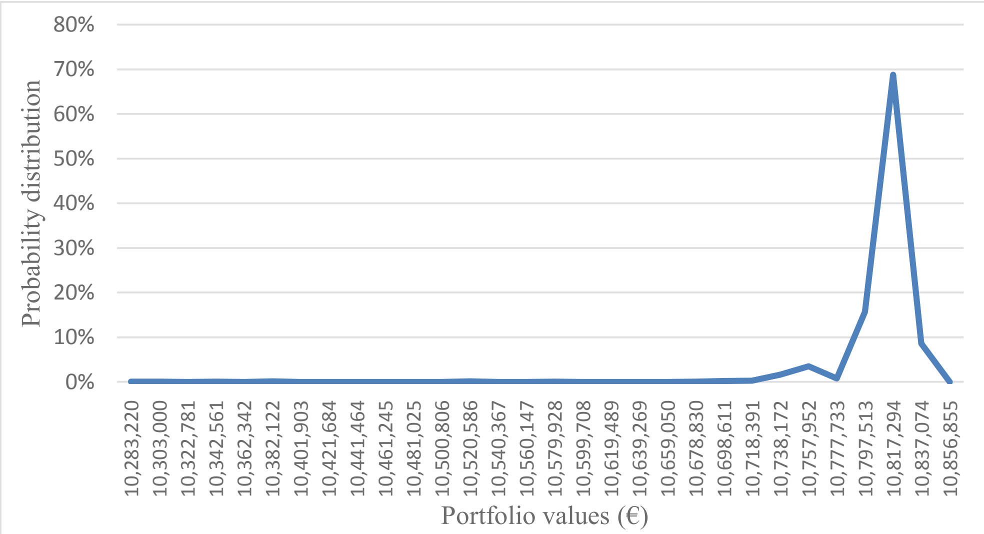
Individual elements of the Cholesky decomposition matrix are calculated according to equations (3.44), (3.45), and (3.46). This matrix allows the consideration of independence due to the fact that random variables of standard normal distribution are generated in the form of vector when multiplied by variables reflecting the relative degree of correlation among the yields of individual issuers. The resulting matrix of correlated random variables can be found in Annex 7.

In the next step, each correlated yield is assigned a credit rating. According to the procedures described in subchapter 3.2.2, it is possible to obtain the limits of the transition among the individual rating categories, which is presents in Annex 8. Besides, the individual correlated yields can be assigned by using the IF function in MS Excel in the respect of the default ratings, which can be found in Annex 9. Once assigning the ratings and knowing the present values of bonds, which are shown in *Tab 4.10*, it is able to use IF function in MS Excel again to check the values of individual bonds. Then, multiplying these values and the number of pieces of bonds included in the portfolio, the total value of bonds by individual issuers can be obtained. The results can be found in Annex 10. When summing up the total value of bonds within each scenario, the value of the overall portfolio is obtained.

4.3.4 Calculation of credit risk

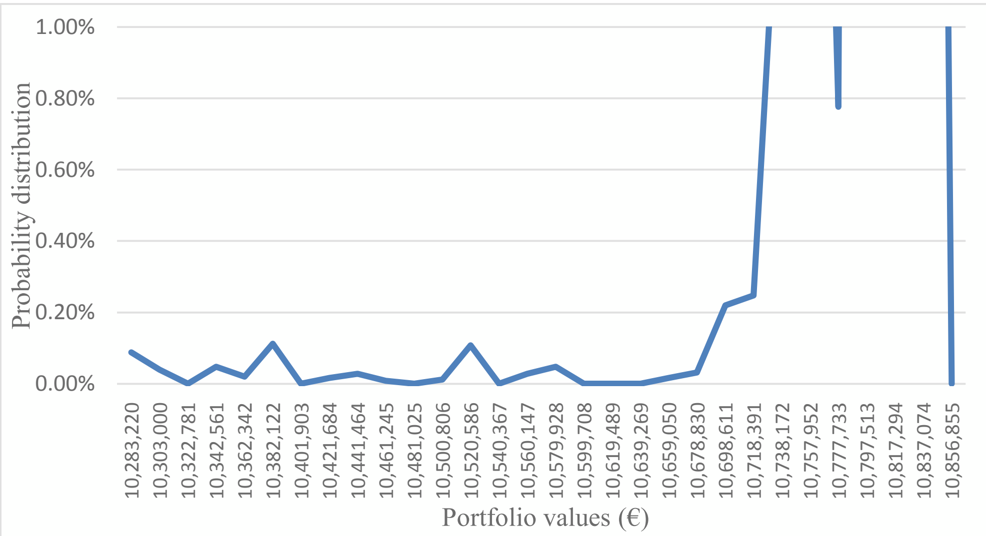
Fourthly, given the fact that all the necessary variables are known according to previous steps, now it is possible to assess the credit risk of the portfolio. *Figure 4.2* below is captured by the probability distribution of the portfolio values. The default values of this figure can be found in Annex 11.

Figure 4.2: Probability distribution of the portfolio values



The probability distribution above illustrates that the portfolio value with a probability of 68.76% is in the range from 10,797,513 € to 10,817,294 €.

Figure 4.3: Probability distribution of the portfolio values – adjusted scale



Credit risk is specifically asymmetrical distribution, which is usually called heavy ends. In other words, most debts are likely to be redeemable and there is only a small probability that debts may cause huge losses. In order to have heavy ends clearly visible, *Figure 4.3* adjust the horizontal axis as above. For example, the portfolio value ranging from 10,698,611 € to 10,718,391 € is with a probability of 0.22%, which is a significant decline. Moreover, the lowest possible portfolio value is 10,263,439 €.

Tab 4.12 below illustrates the value of each bond at their initial ratings and their expected values. The last column, their expected losses, is the difference between the previous two columns.

Tab 4.12: Results of the portfolio value (€)

	Value at initial rating	**Expected value**	**Expected loss**
Deutsche Post	1,105,132	1,101,332	3,799
E.ON	1,173,249	1,172,971	278
Metro	970,857	970,448	409
Volkswagen	1,093,040	1,092,838	202
NIKE	1,069,454	1,069,278	177
Commerzbank	895,744	895,799	-56
Bayer	1,060,761	1,060,684	77
Nestle	1,185,854	1,185,815	39
Danone	1,130,682	1,130,785	-103
Oracle	1,117,341	1,117,108	233
Portfolio	**10,802,113**	**10,797,057**	**5,056**

It is clear that the expected loss of the overall portfolio is 5,056 €, which represents only 0.05% of the total portfolio value. This is mainly because of the high quality of the bonds in the portfolio. Deutsche Post is associated with the highest expected loss, which is 3,799 € and accounts for 75.15% of the total expected loss, because it is highly correlated with other companies in the portfolio, which means its stock prices are more fluctuated. Besides, expected losses of Commerzbank's bond and Danone's bond are negative, which means these two bonds are associated with expected gains.

The parameter of risk is usually expressed in the term of standard deviation, which represent the scatter of values around the mean value. However, given the fact that credit risk has a normal distribution, it is preferable to consider the marginal standard deviation as well. The marginal standard deviation does help to evaluate which assets should be included in the portfolio and which assets should not be. It makes it possible to analyze the effect of each bond on the size of the overall risk. The values of two risk indicators are presented in *Tab 4.13*.

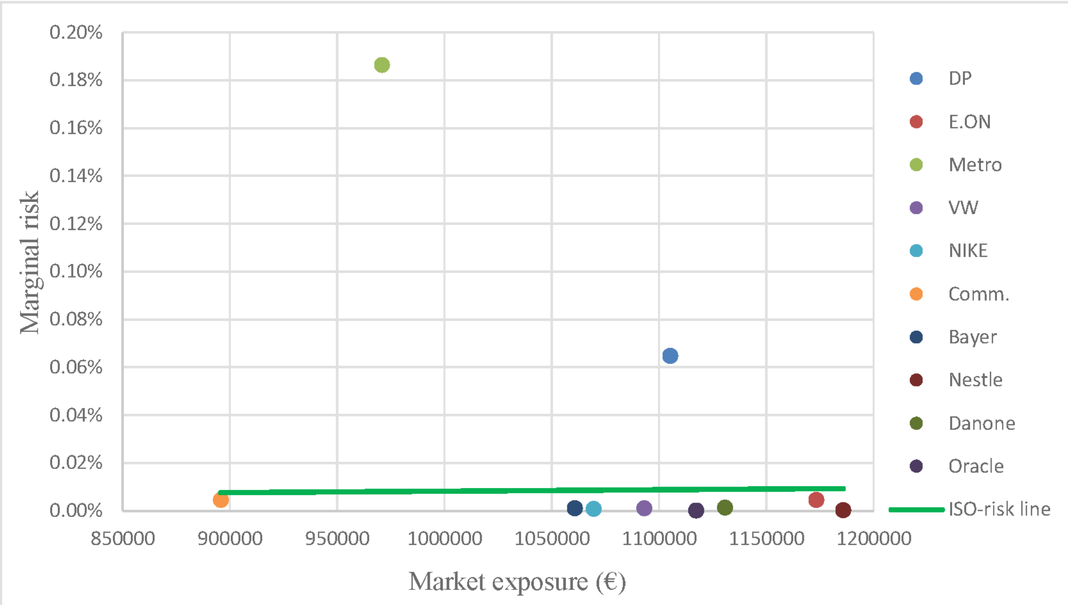
Tab 4.13: Parameters of risk

	Standard deviation		**Marginal standard deviation**	
	%	**€**	**%**	**€**
Deutsche Post	1.2188%	13,423	0.0648%	6,735
E.ON	0.1525%	1,789	0.0046%	479
Metro	2.8056%	27,227	0.1865%	19,382
Volkswagen	0.0778%	850	0.0011%	115
NIKE	0.0772%	826	0.0009%	89
Commerzbank	0.4727%	4,234	0.0047%	484
Bayer	0.0535%	567	0.0011%	112
Nestle	0.0201%	238	0.0003%	36
Danone	0.0986%	1,115	0.0014%	141
Oracle	0.0797%	890	0.0003%	26
Portfolio	**0.3375%**	**35,081**		

The total portfolio risk is relatively low because the standard deviation of the portfolio is only approximately 0.34%. The riskiest bond is issued by Metro with a standard deviation of 2.81% mainly because of the lowest initial rating. Besides, the value of the marginal standard deviation of Metro's bond is highest, too. The fact that marginal standard deviations are lower than standard deviations proves that the diversification has a good effect. The lowest absolute level of risk is associated with the bond issued by Nestle, whose marginal standard deviation is only 0.0003%, because it has the highest initial rating.

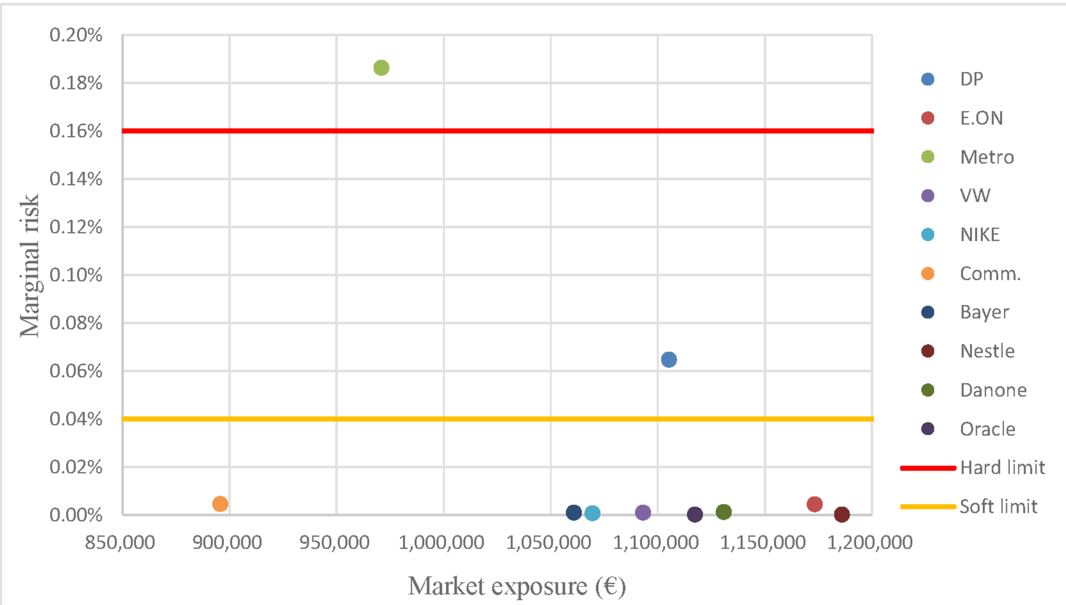
In order to better understand the marginal risk, *Figure 4.5* below concludes marginal risks of all bonds and the ISO-risk line, which consists of points with the same level of the absolute marginal risk. The absolute marginal risk is the product of the market exposure and the marginal standard deviation. The size of this risk corresponds to the median absolute marginal risk of individual bonds, which is equal to 14 € approximately. Bonds located below ISO-risk line are considered less risky than bonds located above it.

Figure 4.5: Marginal risks



It is clear that bonds issued by Deutsche Post and Metro are above the ISO-risk line, while other bonds are below it. The results of analysis of marginal risks can be used to determine the credit limits in *Figure 4.6*.

Figure 4.6: Credit risk limits



Two horizontal lines in *Figure 4.6* are credit risk limits: the orange line represents the soft limit, which is set at 0.04%, and the red line represents the hard limit, which is set at 0.16%. The bonds issued by Deutsche Post and Metro are above the soft limit, while only the bond issued by Metro is above the hard limit. Although there exists another type of limits based on the exposure size, there is no need to consider this type of limits because the exposures to individual issuers are at a very similar level.

The last important parameters are the percentiles, which can determine the economic capitals. These percentiles, namely significance levels, are usually fixed at 0.1%, 0.5%, or 1%. The portfolio values and the unexpected losses (VaR) at different selected percentiles are presented in *Tab 4.14* below.

Tab 4.14: Percentiles and corresponding value of the portfolio and losses

alpha	Portfolio value (€)	VaR (€)
0.1%	10,283,741	-518,372
0.5%	10,559,473	-242,640
1%	10,708,479	-93,634

The obtained results above indicates that the portfolio value and VaR are no greater than 10,283,741 € and 518,372 € respectively at a confidence level at 99.9%. Then it is possible to calculate the economic capital, which represents the amount of capital required to cover unexpected losses from credit risk. The final values of the economic capitals are calculated by the equation (3.17) and concluded in *Tab 4.15*.

Tab 4.15: Percentiles and corresponding economic capitals

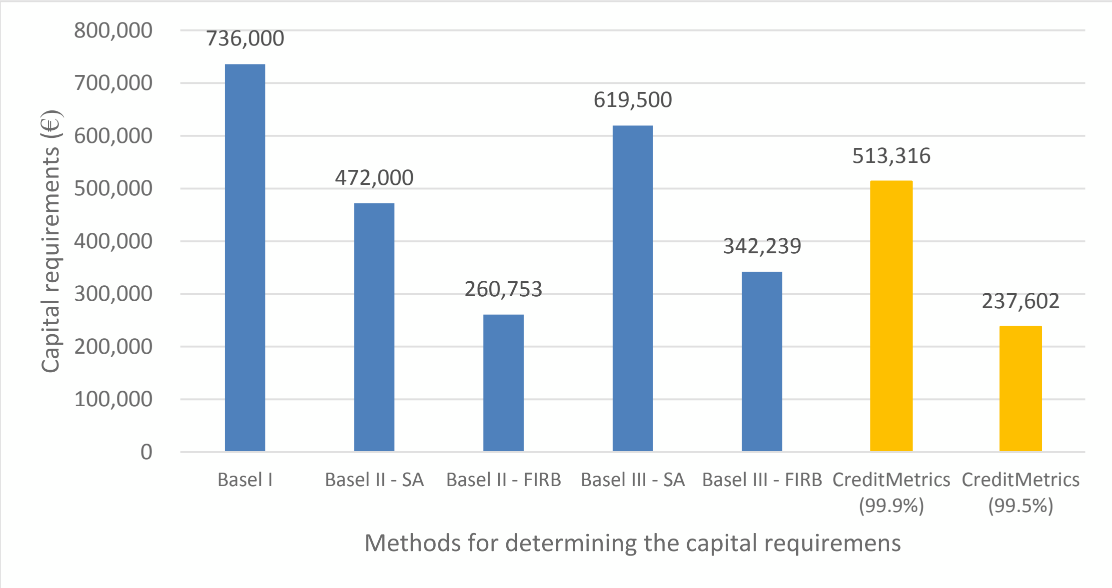
alpha	Economic capital (€)
0.1%	**513,316**
0.5%	**237,602**
1%	88,578

The value of economic capital is 513,316 € at a significance level of 0.01%, while that is 237,602 € at a significance level of 0.05%. Moreover, the values of economic capitals change sharply even when the significance levels change slightly because of the effect of heavy ends typical for credit risk.

4.4 Evaluation of results

In subchapter 4.2, the regulatory capital requirements to cover unexpected losses are calculated under different Basel agreements respectively. And then in subchapter 4.3, the economic capital is obtained by CreditMetrics™ model. The concluded results are presented graphically in *Figure 4.7* below.

Figure 4.7: Regulatory capital requirements under different methods



One of the objectives of the implements of the Basel agreements is to make the regulatory capital requirement as close as possible to the economic capital. The regulatory capital requirement calculated under Basel II in standard approach is closest to the economic capital calculated by the CreditMetrics™ model when the confidence level is 99.9% mainly because these two methods are both based on the Merton model. Besides, when the confidence level is 99.5%, the economic capital, which is equal to 237,602 €, is very close to the regulatory capital requirement calculated by the foundation inter ratings-based approach under both Basel II and Basel III.

It is clear that the value of regulatory capital requirement is lower when the foundation internal ratings-based approach is used than when the standard approach is used. Both under Basel II and Basel III, the values of regulatory capital requirement decrease by 44.76% when using the foundation internal ratings-based approach.

However, the absolute change is greater under Basel III than that under Basel III because the minimum capital adequacy ratio increases by 2.5%.

There are two main reason to explain the different values of the regulatory capital requirement in different methods. One reason is known as the typical granularity that describes a certain diversification and concentration of the portfolio and is built on the foundation internal ratings-based approach. It is a tendency to grant few and large loans or several small loans. Generally speaking, a higher granularity is preferable not only because it prevents the borrowers' excessive concentration in only one or few classes, but also because it allows loan pricing to be more accurate. Therefore, the typical granularity occurs when the portfolio, consisting of only ten selected bonds, is insufficiently diversified. Furthermore, it contributes to an underestimation of risk and a lower regulatory capital requirement.

Another reason for the different values of regulatory capital requirement could be the different degrees of correlation in different methods. The higher the degree of the correlation, the higher the level of the regulatory capital requirement. When using the foundation internal ratings-based approach under both Basel II and Basel III, the correlation (R) is determined by the probability of default and ranges from 22% to 24%, which is a relatively narrow interval. In contrast, when using the CreditMetrics™ model, the correlation presented in the correlation matrix varies from -4% to 72%, which is a wide interval.

5 Conclusion

Nowadays, the importance of financial risks, especially credit risk, is drawing more and more attentions. This is mainly due to the structural changes in economics, namely the increasing level of globalization of financial markets in the world. Credit risk is the potential loss for financial institutions when a borrower will fail to meet its obligations, such as paying interest on the loan and repaying the amount borrowed, in accordance with agreed terms. Therefore, it is important to measure and manage credit risk effectively.

The aim of this thesis was to determine and compare the value of regulatory capital requirement for unexpected losses from credit risk of ten debt assets portfolio under Basel I, Basel II, and Basel III respectively, and the value of economic capital by using the CreditMetrics™ model.

The thesis included both the theoretical and the practical part. The theoretical part described basic financial risks firstly and credit risk was emphasized. Then several complex models, especially the CreditMetrics™ model, for credit risk management were introduced. The last focused on the description of the Basel agreements on capital adequacy. In the practical part, credit risk was calculated with a portfolio consisting of ten selected bonds traded on the Frankfurt Stock Exchange. The nominal value of the overall portfolio was 10 million euro, which meant the nominal value of each bond was 1 million euro, and the time horizon for determining credit risk was one year. The value of regulatory capital requirement to cover unexpected losses was determined under three Basel agreements, both the standard approach and the foundation internal ratings-based approach included. Then the calculation of economic capital of the portfolio by the CreditMetrics™ model was conducted.

One of the objectives of the implements of the Basel agreements is to make the regulatory capital requirement as close as possible to the economic capital. The results obtained in Chapter 4 illustrates that the value of economic capital, which was 513,316 €, calculated by using the CreditMetrics™ model when the confidence level was at 99.9% was similar to the value of regulatory capital requirement, which was 472,000 €, calculated under Basel II by the standard approach. Besides, the economic capital, which was at a confidence level of 99.5% and equal to 237,602 €, was very close to the regulatory capital requirement calculated by the foundation inter ratings-based approach under both Basel II and Basel III. The differences between values probably because of the typical granularity if the portfolio was insufficiently diversified. Another possible reason was the different degrees of correlation in different methods. Specifically speaking, when using the foundation internal ratings-based approach under both Basel II and Basel III, the correlation was determined by the probability of default and ranges from 22% to 24%. In contrast, when using the CreditMetrics™ model, the correlation presented in the correlation matrix varied from -4% to 72%. Moreover, the results also showed that the value of regulatory capital requirement calculated by the foundation internal ratings-based approach was lower than that calculated by the standard approach no matter under Basel II or Basel III, and the relative differences were approximately 44.76%. The absolute change was greater under Basel III than that under Basel III because the minimum capital adequacy ratio increased by 2.5%.

The process of credit risk management is constantly improved because of the changing world and the customers' expectations on financial institutions are rising in line with the changing and advanced technology. Given the fact that the highly complicated financial instruments ultimately depends on the responsibility of each individual investor, it is significant to establish the sufficiently diversified portfolios.

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List of Abbreviations

BCBS	Basel Committee for Banking Supervision
BIS	Bank for International Settlements
CCB	China Construction Bank
CCR	Counterparty credit risk
CR	Capital requirement
CRAs	Credit rating agencies
DD	Distance of default
EAD	Exposure at default
EDF	Expected default frequency
EL	Expected loss
EURIBOR	Euro InterBank Offered Rate
FSE	Frankfurt Stock Exchange
IRS	Interest rate swap
LCR	Liquidity coverage ratio
LDA	Linear discrimination analysis
LIBOR	London InterBank Offered Rate
LGD	Loss given default
LLA	Loan loss allowance
LTD	Long-term debt
NPL	Nonperforming loans
NSFR	Net stable funding ratio
OECD	Organization for Economic Co-operation and Development
PD	Probability of default
PRIBOR	Prague InterBank Offered Rate
RC	Regulatory capital
RR	Recovery rate
RWA	Risk-weighted asset
STD	Short-term debt
UL	Unexpected loss
VaR	Value at Risk

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Student´s name and surname

List of Annexes

Annex 1: Probability matrix from Standard & Poor's

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	85.03%	6.72%	1.52%	0.87%	0.22%	0.43%	0.00%	0.00%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA+	1.09%	74.86%	15.03%	2.73%	0.82%	0.82%	0.55%	0.55%	0.00%	0.27%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.22%	1.20%	78.98%	8.50%	4.14%	1.31%	0.54%	0.22%	0.00%	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA-	0.08%	0.08%	4.56%	74.98%	12.26%	2.73%	1.24%	0.17%	0.08%	0.17%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A+	0.00%	0.07%	0.63%	5.51%	73.97%	10.89%	2.58%	0.49%	0.35%	0.07%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A	0.00%	0.23%	0.17%	0.74%	4.69%	73.46%	11.21%	2.29%	1.14%	0.17%	0.06%	0.06%	0.06%	0.06%	0.00%	0.00%	0.00%	0.11%
A-	0.05%	0.00%	0.16%	0.16%	0.98%	7.22%	76.11%	7.93%	1.48%	0.82%	0.16%	0.05%	0.11%	0.00%	0.00%	0.00%	0.00%	0.05%
BBB+	0.00%	0.00%	0.00%	0.14%	0.29%	0.86%	7.43%	73.50%	8.71%	1.21%	0.36%	0.57%	0.21%	0.21%	0.07%	0.00%	0.14%	0.07%
BBB	0.00%	0.00%	0.10%	0.00%	0.19%	0.58%	0.88%	7.89%	69.98%	7.89%	1.66%	1.07%	0.10%	0.10%	0.39%	0.10%	0.10%	0.10%
BBB-	0.00%	0.00%	0.16%	0.00%	0.16%	0.64%	0.48%	1.43%	8.90%	67.25%	6.52%	2.70%	0.79%	0.32%	0.32%	0.00%	0.32%	0.32%
BB+	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.30%	0.60%	0.90%	11.64%	58.81%	8.06%	2.39%	1.79%	0.30%	0.00%	0.30%	0.00%
BB	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.50%	0.00%	1.75%	11.25%	56.75%	6.25%	2.75%	1.00%	0.00%	0.75%	0.50%
BB-	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.25%	0.25%	8.89%	59.01%	12.84%	4.20%	0.49%	0.25%	1.48%
B+	0.00%	0.00%	0.00%	0.00%	0.00%	0.23%	0.00%	0.23%	0.00%	0.00%	0.23%	2.93%	8.80%	54.63%	8.35%	3.84%	1.35%	1.81%
B	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.38%	0.38%	0.38%	1.51%	12.08%	45.66%	8.30%	4.53%	4.15%
B-	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.63%	0.00%	0.00%	0.00%	0.00%	1.27%	6.33%	49.37%	15.82%	10.13%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.15%	3.46%	9.20%	25.29%	37.93%

Annex 2: Shares prices from March 26th, 2015 to March 16th, 2016 (€)

Date	DP	E.ON	Metro	VW	NIKE	Comm.	Bayer	Nestle	Danone	Oracle
3/26/2015	29.048	14.129	30.550	235.250	91.529	12.845	139.112	69.582	62.060	39.593
3/27/2015	29.140	14.028	31.285	235.900	91.493	12.810	139.422	69.988	62.845	39.574
3/30/2015	29.336	13.991	31.650	241.700	93.615	12.954	142.237	70.900	62.700	40.052
3/31/2015	29.100	13.875	31.595	240.050	94.543	12.830	139.800	70.160	62.710	39.533
4/1/2015	29.135	13.915	32.095	238.950	92.070	13.050	141.650	70.010	63.500	39.545
4/2/2015	29.140	13.741	32.800	237.800	91.748	13.051	139.189	70.134	63.250	40.110
4/3/2015	29.140	13.741	32.800	237.800	91.748	13.051	139.189	70.134	63.250	40.110
4/6/2015	29.140	13.741	32.800	237.800	91.748	13.051	139.189	70.134	63.250	40.110
4/7/2015	29.820	14.299	33.285	238.000	92.200	13.269	141.033	70.853	64.294	40.379
4/8/2015	29.799	14.074	33.450	238.350	92.480	13.275	139.997	71.138	64.152	40.931
4/9/2015	30.685	14.179	34.055	242.000	94.693	13.231	141.844	72.172	64.920	41.428
4/10/2015	31.040	14.289	34.555	244.800	94.701	13.173	145.750	72.837	64.890	41.849
4/13/2015	30.775	14.386	34.305	240.150	94.000	13.231	144.000	72.852	65.000	42.101
4/14/2015	30.792	14.430	34.000	239.350	93.190	13.074	143.340	72.663	64.530	43.006
4/15/2015	30.648	14.560	34.145	236.550	94.220	13.013	142.770	73.756	66.270	43.250
4/16/2015	30.300	14.588	34.040	232.750	92.708	12.575	137.800	74.050	66.570	42.906
4/17/2015	29.501	14.400	33.310	228.600	91.400	12.250	133.550	72.500	66.310	42.595
4/20/2015	30.147	14.489	33.385	227.250	92.783	12.610	136.800	72.110	65.970	41.982
4/21/2015	30.360	14.404	33.590	225.700	93.101	12.400	137.875	72.470	66.310	42.418
4/22/2015	30.039	14.222	32.985	225.850	93.044	12.700	136.500	71.840	66.150	42.129
4/23/2015	29.623	14.219	32.880	226.100	93.562	12.683	134.973	71.640	66.040	42.324
4/24/2015	30.092	14.540	33.300	231.200	92.980	12.765	135.450	71.825	67.050	42.327
4/27/2015	30.549	14.694	33.795	241.500	92.926	12.400	138.038	72.005	67.215	42.523
4/28/2015	29.946	14.560	33.700	236.450	91.100	12.200	134.150	71.245	65.880	42.572
4/29/2015	29.092	14.223	32.780	227.900	89.135	12.082	129.222	69.876	63.969	41.382
4/30/2015	29.477	13.966	32.355	227.300	89.515	12.100	129.731	69.000	64.390	40.340
5/1/2015	29.477	13.966	32.355	227.300	89.515	12.100	129.731	69.000	64.390	40.340
5/4/2015	29.940	14.110	32.765	227.450	90.045	12.159	130.564	70.020	65.270	39.203
5/5/2015	29.072	13.640	32.125	225.100	89.675	11.819	126.615	69.720	64.400	39.448
5/6/2015	29.147	13.676	32.310	217.600	88.050	11.827	127.109	68.200	63.791	39.212
5/7/2015	29.159	13.977	30.665	220.600	89.666	12.212	127.216	69.100	62.175	39.267
5/8/2015	29.910	13.918	30.950	225.350	91.500	12.241	132.830	69.449	63.160	39.911
5/11/2015	29.761	13.969	30.950	225.350	92.400	12.340	132.190	70.000	63.130	41.440
5/12/2015	28.488	13.875	30.775	218.350	91.101	12.351	130.600	69.200	62.390	40.613
5/13/2015	27.690	13.770	30.700	218.150	89.886	12.551	128.730	68.758	62.600	40.578
5/14/2015	28.088	13.890	30.840	220.950	90.008	12.705	130.999	69.000	62.250	39.968
5/15/2015	28.217	13.752	30.975	215.300	91.663	12.520	130.450	68.540	62.330	40.276

5/18/2015	28.460	13.752	31.160	223.050	92.109	12.446	134.000	68.922	62.370	40.241
5/19/2015	28.701	13.979	31.790	232.750	93.884	12.551	137.497	70.298	63.420	40.658
5/20/2015	28.809	14.134	31.885	231.850	94.655	12.640	136.954	70.290	63.450	41.305
5/21/2015	29.069	14.112	32.155	229.600	93.959	12.548	137.726	70.453	63.060	41.011
5/22/2015	28.990	14.049	31.925	229.550	95.003	12.484	137.498	70.628	63.155	40.698
5/25/2015	28.990	14.049	31.925	229.550	95.003	12.484	137.498	70.628	63.155	40.698
5/26/2015	28.850	13.580	31.470	226.250	94.976	12.161	134.921	70.449	64.080	41.620
5/27/2015	29.189	13.681	32.005	228.050	95.128	12.367	137.000	71.092	63.700	40.143
5/28/2015	28.214	13.675	32.140	223.950	93.404	12.230	133.600	71.193	64.108	39.567
5/29/2015	27.550	13.440	31.595	219.500	93.000	12.125	130.232	70.655	63.580	39.193
6/1/2015	27.293	13.389	31.650	219.550	93.489	12.048	131.370	70.650	62.980	39.324
6/2/2015	27.034	13.316	31.465	215.350	91.699	12.130	128.910	69.452	62.940	39.404
6/3/2015	27.076	13.515	32.155	216.250	92.125	12.318	129.448	67.764	63.160	38.409
6/4/2015	26.660	13.080	31.745	215.500	90.660	12.200	127.972	67.400	61.670	37.750
6/5/2015	26.449	13.015	31.550	213.300	90.338	11.946	126.245	66.520	61.000	37.765
6/8/2015	26.172	12.700	31.035	209.800	90.199	11.730	123.745	66.476	61.030	37.255
6/9/2015	25.883	12.650	30.665	208.550	90.049	11.785	124.082	66.028	60.170	36.524
6/10/2015	26.859	12.950	31.235	214.850	91.255	11.939	129.600	66.244	61.290	37.092
6/11/2015	26.735	12.790	31.450	215.300	92.362	11.856	129.712	66.430	61.620	37.543
6/12/2015	26.451	12.688	31.005	212.200	92.200	11.709	127.270	66.589	60.300	38.184
6/15/2015	26.181	12.440	29.560	207.650	91.961	11.450	124.150	65.873	60.270	37.956
6/16/2015	26.297	12.428	29.175	208.150	92.784	11.503	126.990	66.500	61.044	37.585
6/17/2015	25.870	12.280	28.480	206.200	92.502	11.398	124.298	66.201	59.980	38.333
6/18/2015	26.436	12.624	28.840	205.750	93.538	11.511	128.783	65.867	59.709	38.007
6/19/2015	26.181	12.350	28.675	206.350	94.190	11.619	127.471	66.208	59.197	38.200
6/22/2015	26.927	12.685	29.475	215.450	94.054	11.976	133.000	66.668	60.565	38.236
6/23/2015	27.584	12.675	29.785	218.350	95.886	12.129	134.305	67.600	61.045	39.101
6/24/2015	26.969	12.599	30.020	215.750	94.654	11.911	133.240	66.500	60.460	39.219
6/25/2015	27.124	12.422	29.615	215.950	94.350	12.000	133.105	66.361	60.540	39.197
6/26/2015	27.117	12.475	29.840	217.350	98.242	12.120	133.180	67.022	60.320	39.103
6/29/2015	26.320	12.085	29.025	209.400	96.838	11.512	127.370	66.250	59.090	37.478
6/30/2015	26.299	11.990	28.280	207.550	96.976	11.518	126.188	64.450	58.598	37.054
7/1/2015	26.431	11.991	28.640	214.650	98.471	11.748	128.450	65.681	59.253	36.897
7/2/2015	26.370	12.150	28.595	211.450	98.481	11.650	126.622	66.157	59.310	36.206
7/3/2015	26.228	12.102	28.440	210.800	99.094	11.651	126.272	65.600	58.620	36.675
7/6/2015	26.803	11.780	28.055	208.250	99.882	11.203	122.867	65.774	58.090	36.102
7/7/2015	26.610	11.738	27.270	202.950	99.748	10.998	123.060	66.356	57.970	36.602
7/8/2015	26.621	11.720	27.515	197.500	99.400	10.913	123.893	65.710	58.270	36.199
7/9/2015	27.269	11.978	27.890	203.450	100.183	11.076	127.414	67.140	59.065	36.023
7/10/2015	28.088	12.280	28.475	203.400	99.185	11.377	132.850	67.250	61.080	35.696
7/13/2015	28.344	12.369	28.900	205.750	101.873	11.646	135.890	68.256	61.910	35.775

7/14/2015	28.301	12.312	29.300	201.500	102.009	11.595	137.050	68.900	61.800	36.542
7/15/2015	28.481	12.431	29.310	199.150	102.410	11.801	135.982	68.581	61.600	36.524
7/16/2015	28.770	12.570	29.850	203.150	103.060	12.048	137.300	69.531	62.680	36.852
7/17/2015	28.687	12.421	29.800	202.700	103.465	11.970	136.800	69.879	62.557	36.996
7/20/2015	28.962	12.501	29.865	201.000	104.589	12.168	136.290	69.879	62.850	37.040
7/21/2015	28.469	12.389	29.700	198.900	103.734	12.036	133.268	69.358	62.740	37.273
7/22/2015	28.544	12.371	29.645	196.100	104.450	12.010	132.850	68.899	62.679	37.707
7/23/2015	28.318	12.373	29.375	198.500	104.215	12.057	131.411	68.920	62.250	38.079
7/24/2015	27.924	12.161	28.865	189.550	102.996	11.973	130.736	68.426	61.951	38.153
7/27/2015	27.237	11.965	28.260	185.450	99.997	11.620	127.140	66.773	60.370	37.799
7/28/2015	27.628	12.034	28.500	190.600	102.639	11.764	130.834	67.500	60.430	37.597
7/29/2015	27.423	11.977	28.495	185.900	103.540	11.660	135.182	68.507	61.230	37.899
7/30/2015	27.446	12.003	28.350	185.550	105.050	11.732	134.000	68.720	61.240	37.501
7/31/2015	27.563	12.040	28.750	184.050	104.872	11.810	134.209	68.788	61.574	38.074
8/3/2015	27.974	11.997	29.040	185.750	104.000	12.076	134.600	69.350	62.885	37.882
8/4/2015	27.963	12.085	29.280	186.000	105.556	11.910	135.000	68.935	62.910	38.351
8/5/2015	28.246	12.270	29.710	192.400	107.900	11.898	137.016	69.047	63.200	39.135
8/6/2015	27.367	12.315	30.800	190.000	105.041	11.660	136.190	68.500	63.250	38.732
8/7/2015	26.911	12.093	28.975	191.550	103.919	11.534	134.707	67.898	62.720	38.438
8/10/2015	27.258	12.015	28.955	194.150	104.500	11.662	134.850	68.062	62.810	39.419
8/11/2015	26.870	11.752	28.245	186.200	103.660	11.525	131.497	67.300	61.840	38.999
8/12/2015	26.469	11.729	27.155	177.750	100.149	11.243	128.284	66.155	59.880	38.342
8/13/2015	26.451	11.768	27.410	180.450	102.320	11.303	129.950	68.490	60.700	38.056
8/14/2015	26.256	11.677	27.265	180.000	102.470	11.050	129.450	68.868	60.330	38.006
8/17/2015	26.350	11.577	26.970	178.050	103.752	11.058	129.050	69.500	60.030	37.986
8/18/2015	26.171	11.380	26.855	175.200	104.358	10.990	129.348	69.603	60.160	38.268
8/19/2015	25.846	11.131	26.505	171.000	103.478	10.871	125.900	69.282	59.230	37.871
8/20/2015	25.219	10.943	26.195	168.050	100.519	10.430	122.000	68.075	58.350	36.541
8/21/2015	24.451	10.494	25.560	165.000	94.303	10.139	117.180	65.510	56.481	35.753
8/24/2015	24.150	9.864	24.730	158.550	92.716	9.720	115.101	63.680	54.410	34.474
8/25/2015	24.217	10.000	25.650	167.800	93.048	10.069	117.340	64.700	54.820	33.544
8/26/2015	24.472	9.945	25.685	165.950	94.200	9.979	117.330	64.420	54.235	34.983
8/27/2015	24.981	10.293	26.045	171.150	98.050	10.285	122.200	65.759	55.010	36.396
8/28/2015	24.785	10.250	26.065	170.500	100.612	10.136	121.400	65.736	55.138	37.109
8/31/2015	24.590	10.104	26.030	166.700	99.522	10.018	121.300	65.552	55.140	36.829
9/1/2015	23.945	9.893	25.280	161.950	96.280	9.764	116.800	64.549	54.020	35.016
9/2/2015	24.208	9.791	25.370	159.500	97.528	9.829	118.800	64.590	54.330	35.309
9/3/2015	24.650	9.944	26.010	164.400	100.872	10.020	122.250	65.567	55.026	35.705
9/4/2015	24.080	9.449	25.300	159.950	99.000	9.620	117.300	65.088	53.650	35.661
9/7/2015	24.296	9.568	25.395	161.150	98.802	9.763	119.600	65.109	53.540	36.008
9/8/2015	24.399	9.778	25.995	165.400	100.017	10.330	121.284	65.201	53.698	34.911

9/9/2015	24.360	9.631	26.000	169.600	100.016	10.221	120.900	66.096	55.250	35.970
9/10/2015	24.130	8.944	25.725	166.900	98.476	10.221	120.100	66.240	54.030	34.792
9/11/2015	23.955	8.577	25.240	166.250	97.488	10.180	119.299	65.160	53.510	34.681
9/14/2015	23.992	8.589	25.005	165.500	98.778	10.151	119.500	65.313	53.270	35.048
9/15/2015	24.557	8.048	25.070	166.850	101.000	10.158	119.900	66.375	53.430	35.517
9/16/2015	24.665	7.569	25.725	167.500	101.730	10.035	120.100	67.400	54.960	35.997
9/17/2015	24.752	8.140	25.295	167.400	102.023	10.120	119.050	67.377	55.490	34.844
9/18/2015	23.800	7.780	24.810	161.350	100.595	9.741	115.300	67.423	54.610	34.056
9/21/2015	23.770	7.748	24.960	133.700	103.042	9.750	117.910	67.974	55.790	33.634
9/22/2015	23.357	7.489	24.235	111.200	103.725	9.519	114.500	66.172	54.780	33.840
9/23/2015	23.457	7.510	24.240	118.900	103.198	9.407	114.870	65.600	55.300	33.526
9/24/2015	23.419	7.341	23.500	118.900	101.000	9.300	113.434	64.136	54.460	35.829
9/25/2015	24.013	7.420	23.840	115.550	111.750	9.600	116.583	66.409	55.450	36.869
9/28/2015	23.703	7.217	23.430	107.100	107.636	9.295	113.520	65.989	54.850	37.764
9/29/2015	24.500	7.349	23.635	103.300	106.991	9.293	113.000	65.291	54.770	36.250
9/30/2015	24.628	7.680	24.690	104.950	108.991	9.405	114.056	67.159	56.439	36.951
10/1/2015	24.500	7.511	24.360	105.050	109.272	9.280	112.250	66.986	56.650	37.539
10/2/2015	24.949	8.030	24.535	101.150	109.200	9.500	114.002	67.115	56.350	36.947
10/5/2015	25.450	8.342	25.580	102.800	111.591	9.750	116.949	68.070	58.150	37.600
10/6/2015	25.778	8.504	26.125	106.900	108.992	9.824	115.750	68.397	58.214	37.990
10/7/2015	26.009	8.838	26.490	114.900	108.205	10.052	114.650	67.450	57.670	38.455
10/8/2015	26.154	8.872	26.805	116.200	109.689	9.998	111.990	67.942	57.950	38.241
10/9/2015	26.384	9.116	26.920	125.900	109.687	10.070	110.990	67.912	57.920	38.829
10/12/2015	26.059	9.591	26.970	132.450	111.450	9.982	111.150	68.600	58.806	38.086
10/13/2015	25.735	9.140	26.310	130.600	110.680	9.760	109.300	68.740	57.770	38.644
10/14/2015	25.539	9.447	26.070	128.600	110.512	9.669	108.339	67.925	57.760	37.973
10/15/2015	25.780	9.300	26.220	123.800	112.467	9.740	109.150	69.200	59.000	38.131
10/16/2015	25.869	9.483	26.880	121.200	113.800	9.881	109.900	67.752	58.170	38.340
10/19/2015	25.753	9.344	27.540	118.600	117.630	9.935	110.800	67.978	59.405	38.476
10/20/2015	25.706	9.534	27.215	116.250	116.570	9.905	110.240	68.000	58.940	38.840
10/21/2015	26.630	9.455	27.485	119.950	116.431	9.951	109.338	67.702	59.911	38.452
10/22/2015	27.198	9.738	27.905	123.900	117.845	10.124	112.315	69.279	61.820	38.842
10/23/2015	27.572	9.881	28.215	121.850	118.095	10.290	117.055	70.082	62.800	39.646
10/26/2015	27.749	9.920	28.255	125.000	119.417	10.230	116.409	69.336	63.170	39.805
10/27/2015	27.490	9.532	27.990	121.800	118.413	10.148	116.142	69.065	63.010	39.928
10/28/2015	27.347	9.628	28.200	124.050	118.238	10.205	119.500	69.248	63.010	40.608
10/29/2015	27.225	9.515	28.350	125.400	119.748	10.004	120.919	69.594	63.960	41.026
10/30/2015	27.006	9.585	28.030	126.100	119.209	10.060	121.300	69.278	63.520	41.219
11/2/2015	27.182	9.680	28.015	126.750	118.697	10.750	122.851	68.724	63.050	40.922
11/3/2015	27.038	9.741	27.940	124.100	119.850	10.688	122.636	69.350	63.950	40.081
11/4/2015	26.947	9.782	28.145	117.150	120.417	10.624	121.400	70.310	64.480	41.058

11/5/2015	27.186	9.702	28.125	120.100	121.656	10.500	122.754	70.161	64.760	42.213
11/6/2015	27.559	9.681	28.445	121.900	122.740	10.680	123.481	69.638	64.000	42.046
11/9/2015	26.944	9.420	27.975	120.100	121.459	10.581	120.770	69.264	63.700	41.260
11/10/2015	27.081	9.040	27.890	119.750	121.601	10.560	121.201	69.296	63.826	40.760
11/11/2015	26.321	9.288	28.140	118.150	118.718	10.550	122.623	69.360	64.290	41.310
11/12/2015	25.927	8.893	27.630	117.200	117.500	10.330	120.500	69.263	64.180	41.367
11/13/2015	25.750	8.764	27.290	118.450	114.650	10.295	119.833	68.267	62.520	41.647
11/16/2015	26.033	8.782	27.445	116.800	113.838	10.320	120.950	68.102	63.090	41.950
11/17/2015	26.634	8.856	28.890	117.000	115.876	10.365	124.298	69.319	64.510	42.294
11/18/2015	26.588	8.904	29.595	120.000	116.980	10.430	124.162	69.557	63.900	42.725
11/19/2015	26.949	9.149	29.805	125.300	117.265	10.541	125.840	69.662	64.470	43.472
11/20/2015	27.074	9.064	29.645	123.900	123.789	10.515	126.233	69.880	65.000	44.000
11/23/2015	26.956	8.914	29.440	124.650	124.959	10.440	125.420	69.885	64.430	44.155
11/24/2015	26.758	8.838	28.980	127.150	62.287	10.315	123.210	69.005	64.031	44.618
11/25/2015	27.233	9.021	30.665	130.450	63.079	10.367	124.508	69.780	65.430	45.597
11/26/2015	27.614	9.232	31.300	134.550	63.886	10.395	126.850	69.662	65.510	45.385
11/27/2015	27.604	9.020	31.485	134.250	63.550	10.340	125.884	69.985	65.070	45.280
11/30/2015	27.728	8.963	31.560	141.300	62.553	10.390	126.300	70.365	66.230	45.756
12/1/2015	27.153	9.026	29.980	139.650	62.548	10.355	124.989	69.796	65.820	46.412
12/2/2015	26.426	8.961	29.915	137.850	62.730	10.250	123.750	70.475	65.990	46.003
12/3/2015	25.430	8.849	28.925	136.750	60.000	9.901	116.300	69.390	62.730	46.409
12/4/2015	25.800	8.779	28.680	137.300	61.214	10.120	117.966	68.857	62.910	45.093
12/7/2015	25.909	8.791	29.230	136.050	60.574	9.801	118.834	69.245	64.235	45.592
12/8/2015	25.401	8.461	28.535	132.500	61.086	9.545	115.250	68.654	63.510	45.709
12/9/2015	24.762	8.512	28.225	140.400	58.975	9.469	112.155	67.550	62.500	45.195
12/10/2015	25.095	8.541	27.605	139.600	58.907	9.449	114.825	67.813	62.580	45.141
12/11/2015	24.571	8.187	26.720	137.600	57.526	9.220	111.737	66.850	61.200	45.376
12/14/2015	24.480	8.018	27.045	131.700	57.497	9.100	110.600	66.527	61.190	45.329
12/15/2015	25.394	8.279	28.245	134.300	59.116	9.397	113.800	67.311	62.320	44.769
12/16/2015	25.510	8.324	28.470	134.900	59.313	9.369	114.360	67.600	62.495	44.163
12/17/2015	25.994	8.626	29.200	139.150	60.752	9.590	118.250	68.092	63.690	44.990
12/18/2015	25.669	8.385	28.650	140.200	59.743	9.461	115.220	67.727	62.210	45.398
12/21/2015	25.863	8.584	28.735	139.050	59.358	9.395	114.800	66.800	62.420	42.822
12/22/2015	25.719	8.514	28.845	141.000	60.071	9.430	113.400	65.967	60.890	42.991
12/23/2015	25.980	8.952	29.530	143.200	58.974	9.640	116.040	67.908	62.200	42.102
12/24/2015	25.980	8.952	29.530	143.200	58.974	9.640	116.040	67.908	62.200	42.102
12/25/2015	25.980	8.952	29.530	143.200	58.974	9.640	116.040	67.908	62.200	42.102
12/28/2015	26.016	8.899	29.205	142.600	57.954	9.683	115.350	67.900	62.245	41.632
12/29/2015	26.375	8.933	29.610	143.950	58.707	9.689	117.741	69.558	62.930	42.477
12/30/2015	25.998	8.922	29.560	142.300	58.637	9.598	116.165	69.476	63.310	42.630
12/31/2015	25.998	8.922	29.560	142.300	58.637	9.598	116.165	69.476	63.310	42.630

1/1/2016	25.998	8.922	29.560	142.300	58.637	9.598	116.165	69.476	63.310	42.630
1/4/2016	25.385	8.504	28.060	137.850	56.566	9.399	110.827	67.293	60.500	42.519
1/5/2016	25.401	8.489	27.685	130.100	57.657	9.392	111.465	67.860	61.234	43.444
1/6/2016	25.101	8.408	27.580	129.750	57.250	9.200	109.628	67.239	60.440	42.899
1/7/2016	24.110	8.205	26.725	123.550	55.143	8.845	105.900	65.550	59.770	42.984
1/8/2016	24.250	8.121	26.845	125.050	54.908	8.799	104.162	64.927	59.640	42.354
1/11/2016	23.758	8.171	26.530	126.750	54.610	8.650	103.000	64.485	58.837	41.713
1/12/2016	23.969	8.470	27.490	129.350	55.816	8.731	104.290	65.807	60.430	41.350
1/13/2016	23.500	8.598	27.510	129.950	54.999	8.580	103.300	66.017	60.670	42.363
1/14/2016	23.643	8.668	26.945	126.100	53.951	8.501	104.300	65.008	59.360	40.580
1/15/2016	22.518	8.176	26.255	120.350	52.520	8.032	99.500	63.970	59.040	41.272
1/18/2016	22.649	8.231	25.465	118.450	52.984	7.925	101.280	63.878	58.910	40.743
1/19/2016	23.059	8.789	25.825	121.750	54.133	7.948	101.900	64.856	59.660	40.427
1/20/2016	22.256	8.343	25.010	116.050	51.949	7.540	100.251	63.380	57.950	40.004
1/21/2016	22.841	8.727	25.105	122.400	56.237	7.624	103.311	64.309	57.900	39.128
1/22/2016	23.508	8.867	26.530	125.000	56.645	7.900	105.867	65.259	59.870	40.288
1/25/2016	23.264	8.998	26.190	123.000	56.500	7.560	106.105	65.483	59.830	40.136
1/26/2016	22.588	9.411	26.300	124.950	55.813	7.710	105.850	66.020	59.324	40.028
1/27/2016	22.493	9.588	26.500	126.150	55.638	7.678	105.355	66.299	60.760	40.518
1/28/2016	21.889	9.601	25.485	121.550	55.055	7.458	102.200	66.327	61.050	41.079
1/29/2016	22.315	9.429	25.955	120.600	57.311	7.520	103.280	67.569	63.422	40.990
2/1/2016	22.331	9.572	26.180	119.700	57.710	7.490	102.578	67.983	62.945	40.547
2/2/2016	21.810	9.229	25.675	117.650	57.986	7.130	99.668	67.603	63.000	41.493
2/3/2016	21.177	9.133	24.820	113.950	55.707	6.949	98.567	67.489	62.105	41.064
2/4/2016	20.897	9.358	25.120	115.300	54.118	7.165	98.633	66.795	62.160	40.471
2/5/2016	20.988	9.490	25.665	118.850	51.701	7.310	97.152	66.345	61.030	39.364
2/8/2016	20.031	9.041	25.150	111.700	49.333	6.683	94.000	65.416	60.030	39.651
2/9/2016	19.972	8.725	24.555	109.500	48.989	6.348	92.618	65.149	60.180	37.375
2/10/2016	20.500	8.467	24.690	111.300	51.300	6.852	94.872	66.331	60.365	36.572
2/11/2016	19.826	8.203	23.055	108.950	49.432	6.499	93.425	64.200	58.463	37.298
2/12/2016	19.870	8.446	23.145	110.600	49.936	7.615	94.895	64.938	58.600	35.752
2/15/2016	20.922	9.061	23.750	118.650	51.436	7.539	96.526	66.661	60.179	37.999
2/16/2016	21.299	8.908	23.525	117.750	51.649	7.424	96.000	66.724	60.400	38.300
2/17/2016	21.628	8.840	24.045	123.450	52.092	7.511	98.654	66.971	60.560	38.834
2/18/2016	21.510	8.906	24.200	125.150	52.975	7.439	99.365	64.631	60.630	40.441
2/19/2016	21.029	8.569	23.800	119.450	52.280	7.265	98.200	64.299	60.169	40.147
2/22/2016	21.862	8.990	23.535	122.950	54.548	7.490	99.130	64.650	60.890	42.537
2/23/2016	21.494	8.632	23.065	120.800	54.811	7.396	97.360	64.399	63.400	42.557
2/24/2016	21.087	8.357	21.870	117.000	54.061	7.149	95.919	63.701	62.270	43.612
2/25/2016	22.009	8.525	22.195	116.600	56.168	7.250	95.550	63.966	64.170	44.137
2/26/2016	22.066	8.520	22.770	122.250	57.394	7.400	97.955	64.839	64.540	44.162

2/29/2016	21.911	8.402	22.695	128.800	57.233	7.499	96.370	65.050	63.500	44.177
3/1/2016	22.231	8.762	23.410	131.000	57.075	7.631	98.350	65.945	64.310	45.095
3/2/2016	22.327	8.633	23.635	134.200	57.250	7.958	99.150	65.412	62.950	45.938
3/3/2016	22.493	8.615	23.725	135.300	55.721	8.060	98.100	65.144	63.250	45.760
3/4/2016	22.788	8.688	24.350	138.500	55.383	8.120	99.100	65.235	63.560	45.770
3/7/2016	22.652	8.560	24.165	135.750	54.062	8.179	98.574	65.037	63.670	45.813
3/8/2016	23.664	8.376	24.425	130.300	55.000	8.019	98.000	64.992	62.600	46.314
3/9/2016	22.982	8.128	24.715	130.450	52.976	8.031	98.468	65.189	63.950	46.082
3/10/2016	22.605	8.020	23.965	125.250	51.813	8.040	95.200	64.168	62.500	45.742
3/11/2016	23.467	8.329	24.800	130.450	53.871	8.415	98.130	65.127	63.830	44.749
3/14/2016	23.916	8.590	25.000	132.400	54.736	8.489	100.450	65.543	64.200	45.691
3/15/2016	23.975	8.455	24.630	128.300	55.175	8.350	100.280	65.294	63.620	45.927
3/16/2016	24.055	8.471	24.740	130.700	55.380	8.305	100.300	65.266	63.060	46.241

Annex 3: Covariance matrix

	Deutsche Post	E.ON	Metro	VW	NIKE	Comm.	Bayer	Nestle Holdings	Danone	Oracle
Deutsche Post	0.00030	0.00021	0.00020	0.00022	0.00023	0.00021	0.00024	0.00011	0.00014	0.00005
E.ON	0.00021	0.00057	0.00025	0.00032	0.00018	0.00028	0.00025	0.00012	0.00015	0.00003
Metro	0.00020	0.00025	0.00038	0.00026	0.00019	0.00022	0.00024	0.00013	0.00018	0.00004
VW	0.00022	0.00032	0.00026	0.00086	0.00010	0.00029	0.00023	0.00013	0.00013	0.00008
NIKE	0.00023	0.00018	0.00019	0.00010	0.00137	0.00021	0.00024	0.00017	0.00016	0.00006
Comm.	0.00021	0.00028	0.00022	0.00029	0.00021	0.00053	0.00026	0.00012	0.00012	-0.00002
Bayer	0.00024	0.00025	0.00024	0.00023	0.00024	0.00026	0.00035	0.00014	0.00017	0.00005
Nestle Holdings	0.00011	0.00012	0.00013	0.00013	0.00017	0.00012	0.00014	0.00014	0.00012	0.00004
Danone	0.00014	0.00015	0.00018	0.00013	0.00016	0.00012	0.00017	0.00012	0.00021	0.00006
Oracle	0.00005	0.00003	0.00004	0.00008	0.00006	-0.00002	0.00005	0.00004	0.00006	0.00032

Annex 4: Yield curves derived from the annual transition matrix

1st year: 2016

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	72.38%	10.76%	3.54%	1.72%	0.60%	0.81%	0.11%	0.07%	0.35%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA+	1.78%	56.30%	23.27%	5.43%	2.23%	1.63%	1.10%	0.92%	0.10%	0.42%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.38%	1.87%	62.98%	13.36%	7.45%	2.73%	1.22%	0.45%	0.07%	0.19%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA-	0.14%	0.19%	7.12%	57.31%	18.59%	5.54%	2.54%	0.49%	0.24%	0.28%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A+	0.01%	0.14%	1.25%	8.35%	55.96%	16.41%	5.20%	1.22%	0.72%	0.18%	0.02%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%
A	0.01%	0.35%	0.38%	1.40%	7.13%	55.34%	17.08%	4.37%	2.03%	0.46%	0.14%	0.13%	0.11%	0.09%	0.01%	0.00%	0.01%	0.20%
A-	0.08%	0.02%	0.28%	0.37%	1.86%	10.99%	59.37%	12.16%	3.01%	1.42%	0.33%	0.18%	0.18%	0.04%	0.02%	0.00%	0.02%	0.11%
BBB+	0.00%	0.00%	0.03%	0.24%	0.58%	1.89%	11.31%	55.34%	12.73%	2.51%	0.78%	0.93%	0.37%	0.34%	0.16%	0.04%	0.16%	0.20%
BBB	0.00%	0.00%	0.17%	0.04%	0.35%	1.04%	1.99%	11.53%	50.40%	11.15%	2.80%	1.76%	0.33%	0.29%	0.52%	0.17%	0.18%	0.27%
BBB-	0.00%	0.00%	0.25%	0.03%	0.29%	1.02%	0.98%	2.82%	12.41%	46.76%	8.68%	4.06%	1.37%	0.74%	0.51%	0.08%	0.37%	0.71%
BB+	0.00%	0.00%	0.02%	0.00%	0.03%	0.11%	0.53%	1.10%	2.26%	14.90%	36.28%	9.91%	3.58%	2.64%	0.70%	0.13%	0.40%	0.27%
BB	0.00%	0.00%	0.00%	0.00%	0.01%	0.04%	0.41%	0.77%	0.32%	3.51%	13.14%	33.80%	7.78%	4.20%	1.58%	0.29%	0.75%	1.26%
BB-	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.03%	0.10%	0.35%	0.54%	1.36%	10.71%	36.58%	15.36%	5.60%	1.40%	0.72%	2.95%
B+	0.00%	0.00%	0.00%	0.00%	0.01%	0.30%	0.05%	0.32%	0.07%	0.13%	0.65%	4.10%	10.32%	32.13%	9.06%	4.85%	2.11%	4.19%
B	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.00%	0.04%	0.09%	0.48%	0.50%	0.92%	2.68%	12.48%	22.61%	8.78%	4.70%	8.85%
B-	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.05%	0.75%	0.07%	0.04%	0.07%	0.21%	2.27%	6.67%	26.40%	12.12%	21.42%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.06%	0.01%	0.02%	0.05%	0.15%	1.45%	3.13%	7.20%	8.02%	48.62%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

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10th year: 2025

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	17.63%	9.04%	11.12%	6.44%	5.11%	3.78%	2.54%	1.30%	0.81%	0.44%	0.12%	0.08%	0.04%	0.03%	0.02%	0.01%	0.01%	0.07%
AA+	1.75%	5.76%	15.60%	10.80%	9.89%	7.13%	5.31%	2.64%	1.36%	0.77%	0.23%	0.17%	0.09%	0.07%	0.04%	0.01%	0.02%	0.15%
AA	0.57%	1.43%	11.94%	11.17%	11.66%	8.85%	6.52%	2.94%	1.50%	0.74%	0.23%	0.16%	0.09%	0.07%	0.03%	0.01%	0.02%	0.16%
AA-	0.26%	0.61%	5.80%	10.61%	13.12%	11.34%	8.91%	4.11%	2.16%	1.00%	0.33%	0.24%	0.13%	0.10%	0.05%	0.02%	0.02%	0.25%
A+	0.11%	0.35%	2.61%	5.90%	10.61%	12.15%	10.96%	5.57%	2.99%	1.34%	0.46%	0.34%	0.19%	0.15%	0.08%	0.03%	0.04%	0.40%
A	0.09%	0.28%	1.25%	2.63%	5.76%	11.18%	13.30%	7.99%	4.48%	2.11%	0.78%	0.59%	0.33%	0.26%	0.14%	0.06%	0.06%	0.94%
A-	0.11%	0.16%	0.74%	1.49%	3.61%	8.63%	14.05%	9.99%	5.86%	2.91%	1.12%	0.84%	0.46%	0.37%	0.20%	0.09%	0.09%	0.99%
BBB+	0.04%	0.06%	0.37%	0.72%	1.79%	4.54%	8.95%	10.20%	7.42%	4.00%	1.66%	1.25%	0.67%	0.57%	0.32%	0.15%	0.14%	1.70%
BBB	0.02%	0.04%	0.28%	0.40%	0.98%	2.39%	4.64%	6.73%	7.14%	4.94%	2.22%	1.61%	0.88%	0.75%	0.41%	0.20%	0.17%	2.31%
BBB-	0.01%	0.03%	0.26%	0.30%	0.69%	1.58%	2.75%	4.09%	5.23%	5.08%	2.60%	1.94%	1.18%	1.02%	0.52%	0.27%	0.21%	3.54%
BB+	0.00%	0.01%	0.10%	0.11%	0.28%	0.73%	1.33%	2.07%	2.89%	3.59%	2.33%	1.94%	1.37%	1.25%	0.61%	0.34%	0.23%	3.66%
BB	0.00%	0.01%	0.05%	0.05%	0.14%	0.39%	0.73%	1.10%	1.48%	2.16%	1.76%	1.74%	1.45%	1.39%	0.68%	0.41%	0.25%	5.96%
BB-	0.00%	0.00%	0.02%	0.02%	0.06%	0.18%	0.30%	0.47%	0.64%	1.05%	1.15%	1.61%	1.86%	1.99%	1.03%	0.71%	0.39%	12.04%
B+	0.00%	0.00%	0.01%	0.02%	0.07%	0.18%	0.26%	0.34%	0.39%	0.59%	0.66%	1.01%	1.26%	1.47%	0.80%	0.62%	0.33%	15.34%
B	0.00%	0.00%	0.01%	0.01%	0.02%	0.08%	0.11%	0.18%	0.23%	0.32%	0.34%	0.53%	0.69%	0.89%	0.53%	0.46%	0.25%	22.09%
B-	0.00%	0.00%	0.00%	0.01%	0.02%	0.05%	0.08%	0.15%	0.19%	0.16%	0.13%	0.20%	0.27%	0.41%	0.29%	0.32%	0.17%	40.74%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.05%	0.05%	0.05%	0.08%	0.12%	0.17%	0.12%	0.12%	0.07%	56.98%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Annex 5: Forward yield curves from 2016 to 2025

	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
AAA	-0.16%	-0.19%	-0.01%	0.23%	0.51%	0.82%	1.02%	1.39%	1.49%	1.69%
AA+	-0.16%	-0.19%	-0.01%	0.23%	0.52%	0.82%	1.03%	1.39%	1.49%	1.70%
AA	-0.16%	-0.19%	-0.01%	0.23%	0.52%	0.82%	1.03%	1.39%	1.50%	1.70%
AA-	-0.16%	-0.19%	-0.01%	0.23%	0.52%	0.82%	1.03%	1.39%	1.50%	1.70%
A+	-0.15%	-0.18%	0.00%	0.24%	0.53%	0.83%	1.04%	1.40%	1.51%	1.71%
A	-0.06%	-0.12%	0.05%	0.28%	0.56%	0.87%	1.07%	1.43%	1.54%	1.74%
A-	-0.11%	-0.15%	0.03%	0.27%	0.56%	0.86%	1.07%	1.43%	1.54%	1.74%
BBB+	-0.06%	-0.10%	0.08%	0.31%	0.60%	0.90%	1.11%	1.47%	1.58%	1.78%
BBB	-0.02%	-0.07%	0.11%	0.35%	0.64%	0.94%	1.14%	1.51%	1.61%	1.81%
BBB-	0.19%	0.08%	0.23%	0.46%	0.73%	1.03%	1.22%	1.58%	1.68%	1.88%
BB+	-0.03%	-0.03%	0.17%	0.42%	0.71%	1.02%	1.22%	1.59%	1.69%	1.88%
BB	0.46%	0.31%	0.44%	0.65%	0.92%	1.20%	1.39%	1.74%	1.82%	2.01%
BB-	1.32%	0.94%	1.01%	1.18%	1.42%	1.67%	1.83%	2.15%	2.21%	2.37%
B+	1.98%	1.52%	1.53%	1.64%	1.82%	2.02%	2.13%	2.41%	2.44%	2.58%
B	4.58%	3.30%	2.92%	2.77%	2.75%	2.81%	2.81%	3.00%	2.96%	3.04%
B-	13.14%	9.31%	7.65%	6.62%	5.96%	5.54%	5.17%	5.08%	4.81%	4.70%
CCC	46.01%	23.90%	16.53%	12.86%	10.74%	9.40%	8.42%	7.89%	7.28%	6.91%

Annex 6: Random variables

	$\tilde{\varepsilon}_{DP}$	$\tilde{\varepsilon}_{E.ON}$	$\tilde{\varepsilon}_{Metro}$	$\tilde{\varepsilon}_{VW}$	$\tilde{\varepsilon}_{NIKE}$	$\tilde{\varepsilon}_{Comm.}$	$\tilde{\varepsilon}_{Bayer}$	$\tilde{\varepsilon}_{Nestle}$	$\tilde{\varepsilon}_{Danone}$	$\tilde{\varepsilon}_{Oracle}$
1	-1.4412	-0.3459	0.8033	-0.4339	0.4073	-1.2629	-1.1118	0.1114	0.0429	-0.3741
2	1.6791	-0.8605	0.6485	0.3151	1.3014	-0.5075	0.8990	-0.5063	0.7086	-0.9904
3	0.1063	0.0852	-2.5490	-0.8680	0.0073	-0.1904	-0.8478	-0.0614	0.5383	0.7161
4	-1.5314	-1.2648	0.1899	0.3724	0.4665	-0.7375	1.7189	-1.1990	0.5174	-1.5095
5	-0.6348	-0.4419	1.8759	0.2211	-0.1447	-0.5988	0.0451	-0.5978	1.4289	0.1978
6	-0.7195	0.3223	-0.9305	-0.1674	-0.7318	0.6791	-1.5314	-0.9284	-0.7582	0.9033
7	0.3251	0.1170	1.2067	0.3218	-0.0535	-1.0088	0.1488	-1.1926	-0.1831	-0.8075
8	-0.7471	-1.6298	0.0762	2.3593	-0.9522	0.5067	-0.8418	-0.9782	-0.8666	0.6318
9	0.0837	-0.2310	1.0086	-0.5975	0.0491	-0.1904	0.0569	-1.2528	-0.5858	0.9825
10	1.2498	-1.1857	2.4579	-1.8202	-0.0067	-1.3720	0.4642	-0.0820	1.9113	0.2741
11	-0.0845	-0.1422	0.8502	0.0357	0.0849	1.6121	2.1223	-0.0436	-0.3663	0.0314
12	0.0399	-0.9227	-0.1768	0.5064	-1.0673	0.4613	-1.2201	-1.0720	-0.7655	-0.0781
13	0.6940	1.0008	-1.0307	0.8444	0.7900	0.0638	-0.6777	0.4430	1.0387	0.8504
14	-0.0163	-0.7440	0.6841	-1.8853	-1.8427	1.1352	-0.6171	-0.0997	1.1058	0.9511
15	0.0957	0.5171	0.2660	-0.0116	-0.9212	2.2163	0.5996	-0.4930	0.7407	-0.3138
16	-0.8193	-0.0906	1.1761	-0.5765	1.8693	-0.8636	-0.6159	-0.0623	-1.9575	2.1065
17	-0.1249	-1.7172	0.6141	-0.2331	-0.6818	-0.4184	-1.5520	1.6332	0.0283	-0.1198
18	-0.5029	0.0378	0.3817	-0.6674	-1.2285	2.2961	-1.1443	-0.0085	-0.7937	0.1461
19	0.2492	-1.4260	1.1905	-1.1854	0.9096	1.3237	-0.3775	0.0802	2.3423	-0.0125
20	-0.7025	-1.9957	0.5699	0.6017	-0.4659	-0.0442	-0.8715	1.2448	0.1469	1.9743
21	0.9143	-0.0026	0.9528	-0.7455	1.1520	1.1108	-0.8027	-0.3407	-0.2498	-1.3459
22	-0.6408	0.5480	1.9930	-0.3963	-0.8743	1.1019	0.0486	-1.1694	-0.4154	1.7030
23	0.7618	-0.3775	1.0095	-0.8299	0.4271	-1.6853	1.5014	0.5085	-1.2377	-0.3098
24	0.6165	0.2273	0.0830	0.9661	2.1439	0.1557	0.4853	-0.2883	0.3090	0.6478
25	1.3608	-0.5651	0.1301	1.7155	0.1744	-0.1410	0.2536	0.1115	0.5613	2.1673
26	2.5671	0.0309	0.6345	1.9247	-0.4954	-0.4125	-1.2848	1.0447	-0.6158	-0.8679
27	0.8422	0.3340	-0.4835	0.9711	0.7442	-0.4038	-0.3753	-2.1201	0.9570	0.8934
28	0.2120	0.3404	-0.3789	-0.6279	0.8510	-0.2379	-0.4674	0.7962	-1.2184	-0.2449
29	1.3372	-0.7879	-0.2031	-1.0147	-0.6406	-0.1632	0.3850	0.4014	0.2514	0.3858
30	-1.5604	-1.8494	-0.3755	0.3724	0.1885	-1.0016	-0.1585	0.1886	0.1342	3.8417
31	-0.4148	-0.5017	1.1428	1.8541	-0.8745	1.0830	0.7084	-1.5252	-0.1121	1.6240
32	-0.6500	-1.3169	-0.2488	-0.8929	0.6532	0.6552	-2.0654	1.9874	-0.2454	0.7732
33	0.0685	0.2614	-1.4967	-0.9384	-1.1036	0.1817	-0.4946	-0.2076	-0.7990	0.6105
34	-0.7429	0.9281	0.0587	-0.9058	0.8265	-1.4165	0.4733	0.1587	-0.8260	1.0110
35	-0.0451	0.3990	-1.2662	0.4545	-0.6335	1.4479	-1.1403	-1.7043	1.8138	2.3147
Etc.	…	…	…	…	…	…	…	…	…	…

Annex 7: Correlated random variables

	$\tilde{\varepsilon}_{DP}$	$\tilde{\varepsilon}_{E.ON}$	$\tilde{\varepsilon}_{Metro}$	$\tilde{\varepsilon}_{VW}$	$\tilde{\varepsilon}_{NIKE}$	$\tilde{\varepsilon}_{Comm.}$	$\tilde{\varepsilon}_{Bayer}$	$\tilde{\varepsilon}_{Nestle}$	$\tilde{\varepsilon}_{Danone}$	$\tilde{\varepsilon}_{Oracle}$
1	-2.6428	-0.7440	0.1943	-0.5809	0.2141	-1.0718	-0.6792	0.0518	-0.0030	-0.3546
2	2.5538	-0.3502	0.8064	-0.0341	1.1732	-0.1317	0.4816	-0.2815	0.3776	-0.9389
3	-2.1025	-0.9512	-2.0675	-0.7436	-0.0200	-0.4276	-0.3660	0.1954	0.4076	0.6788
4	-1.4986	-0.8568	0.2780	0.0182	0.2916	-0.1196	0.7728	-0.9104	0.2111	-1.4310
5	0.5297	0.1541	1.7122	0.0386	-0.1645	-0.5786	0.1810	-0.0004	0.9419	0.1876
6	-2.9405	-0.4591	-1.4790	0.0474	-0.9602	0.1235	-1.1373	-0.8053	-0.4169	0.8563
7	-0.0605	0.0205	0.4311	0.0657	-0.3638	-0.6618	-0.2292	-1.0300	-0.1850	-0.7655
8	-2.0507	-1.1360	-0.2733	2.1680	-1.1436	0.1424	-0.7729	-0.9055	-0.5094	0.5989
9	-0.5449	-0.4817	0.1365	-0.4669	-0.1900	-0.3555	-0.1989	-0.9858	-0.2987	0.9313
10	1.9683	-0.8361	2.1230	-1.7651	0.0878	-1.1267	0.6157	0.5318	1.2606	0.2598
11	2.5806	0.9076	1.1682	0.2516	0.3269	1.5700	1.2157	-0.1361	-0.2347	0.0298
12	-2.3254	-1.1686	-0.8631	0.5420	-1.3275	0.1739	-1.0667	-1.0273	-0.5023	-0.0741
13	1.7265	0.9767	-0.2113	0.6586	0.8720	-0.2260	-0.0755	0.7311	0.7427	0.8062
14	-0.5916	-0.6587	0.5303	-1.3079	-1.5978	0.5688	-0.1154	0.3607	0.7945	0.9016
15	1.9022	1.2804	0.6901	0.2241	-0.7246	1.8154	0.3709	-0.1839	0.4540	-0.2975
16	-1.3999	-0.6664	0.0277	-0.4680	1.5732	-1.0747	-0.5215	-0.3769	-1.0948	1.9969
17	-1.4692	-1.5479	0.4686	-0.1450	-0.4974	-0.4556	-0.6518	1.2036	0.0085	-0.1136
18	-0.9921	0.1691	-0.0762	-0.1695	-1.1594	1.6198	-0.8144	-0.2223	-0.5021	0.1385
19	1.7841	-0.4812	1.7172	-1.0857	1.0727	0.8875	0.1827	0.7452	1.5163	-0.0119
20	-0.8618	-1.4063	0.7731	0.7316	-0.1864	-0.4727	-0.0968	1.1930	0.2575	1.8716
21	1.0313	0.1299	0.4403	-0.7337	0.9266	0.9865	-0.7128	-0.4812	-0.2724	-1.2758
22	0.4358	0.8542	1.1424	-0.0285	-0.9140	0.5119	-0.0935	-0.7907	-0.1291	1.6144
23	0.6861	-0.5806	0.4178	-0.8144	0.4205	-0.9602	0.7599	-0.0225	-0.8272	-0.2937
24	2.5253	0.6683	0.5156	0.6658	2.0423	0.0476	0.3539	-0.0479	0.2535	0.6141
25	2.8118	0.1107	0.7108	1.5601	0.3316	-0.4874	0.4684	0.4979	0.5418	2.0545
26	2.5596	0.3920	0.4836	1.6117	-0.4875	-0.2766	-0.7740	0.4924	-0.4702	-0.8227
27	0.4819	0.0608	-0.5159	0.6204	0.3531	-0.6690	-0.3617	-1.1855	0.6934	0.8469
28	-0.5721	-0.2003	-0.6793	-0.5469	0.7844	-0.1236	-0.3693	0.2036	-0.8094	-0.2321
29	0.7419	-0.8748	-0.0889	-0.7735	-0.4627	-0.1324	0.3829	0.4155	0.1945	0.3657
30	-2.3278	-1.8973	-0.1641	0.4950	0.3147	-1.5038	0.3151	0.6233	0.4028	3.6418
31	1.0329	0.4723	0.9960	1.8326	-0.9047	0.5801	0.2857	-0.9743	0.0609	1.5395
32	-1.7394	-1.3812	-0.1315	-0.6111	0.8465	0.1662	-0.8628	1.4888	-0.0954	0.7330
33	-2.2057	-0.6775	-1.7391	-0.5915	-1.1274	-0.0220	-0.4155	-0.3176	-0.4674	0.5788
34	-0.9483	0.1475	-0.3678	-0.8382	0.7406	-1.1772	0.2675	-0.0079	-0.4519	0.9584
35	-0.1909	0.3061	-0.7072	0.5703	-0.6857	0.3820	-0.4731	-0.4619	1.3653	2.1943
Etc.	…	…	…	…	…	…	…	…	…	…

Annex 8: Breakpoints

Rating	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC
AAA	1.646	1.841	1.667	1.793			x										
AA+	-1.283	1.709	1.645	1.783	1.604	1.594											
AA	-1.844	-0.814	1.538	1.773	1.597	1.574	1.668		x	x							
AA-	-2.111	-1.577	-1.044	1.381	1.543	1.559	1.652	1.536									
A+	-2.378	-1.879	-1.528	-0.968	1.193	1.499	1.636	1.524	1.343	1.290							
A	-2.484	-2.016	-2.018	-1.707	-1.063	1.207	1.549	1.502	1.331	1.281				0.931			
A-		-2.206	-2.378	-2.130	-1.813	-1.030	1.113	1.438	1.297	1.245	1.040	0.906					
BBB+		-2.400	-2.716	-2.636	-2.362	-1.757	-1.248	1.039	1.247	1.220	1.027	0.896		0.922			
BBB	x			-2.807	-2.636	-2.130	-1.932	-1.198	0.896	1.147	1.002		1.170			x	
BBB-		x	x	-2.929	-3.195	-2.562	-2.260	-1.905	-1.200	0.791	0.966	0.878	1.158		0.751		
BB+						-2.697	-2.678	-2.137	-1.797	-1.211	0.572	0.815	1.146	0.913	0.739		
BB						-2.759	-2.863	-2.235	-2.062	-1.668	-1.134	0.468	1.134	0.904	0.726		
BB-						-2.834	-2.948	-2.457	-2.370	-2.040	-1.667	-1.213	0.781	0.799	0.714		0.740
B+						-2.929		-2.583	-2.414	-2.232	-1.979	-1.645	-0.868	0.524	0.666	0.951	0.740
B								-2.770	-2.462	-2.342	-2.512	-2.005	-1.520	-1.022	0.322	0.902	0.702
B-									-2.748				-2.010	-1.476	-0.955	0.685	0.595
CCC								-2.863	-2.878	-2.489	x	-2.241	-2.113	-1.858	-1.361	-0.645	0.338
D						x	x	-3.195	-3.090	-2.727		-2.576	-2.175	-2.095	-1.734	-1.274	-0.307

Annex 9: Rating assignments

	DP	E.ON	Metro	VW	NIKE	Comm.	Bayer	Nestle	Danone	Oracle
Default	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
Scenario										
1	BBB-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
2	AA+	A-	BBB	A+	AA-	BBB+	A-	AA	BBB+	A+
3	BBB	A-	BB-	A+	AA-	BBB+	A-	AA	BBB+	A+
4	BBB+	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A
5	A-	A-	AA-	A+	AA-	BBB+	A-	AA	BBB+	A+
6	BB	A-	BB+	A+	AA-	BBB+	A-	AA	BBB+	A+
7	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
8	BBB	A-	BBB-	AAA	A+	BBB+	A-	AA	BBB+	A+
9	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
10	AA+	A-	AA-	A	AA-	BBB+	A-	AA	A-	A+
11	AA+	A-	BBB+	A+	AA-	AA	A	AA	BBB+	A+
12	BBB-	A-	BBB-	A+	A+	BBB+	A-	AA	BBB+	A+
13	AA+	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
14	A-	A-	BBB-	A	A+	BBB+	A-	AA	BBB+	A+
15	AA+	A	BBB-	A+	AA-	AA	A-	AA	BBB+	A+
16	BBB+	A-	BBB-	A+	AA	BBB+	A-	AA	BBB+	AAA
17	BBB+	BBB+	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
18	A-	A-	BBB-	A+	A+	AA	A-	AA	BBB+	A+
19	AA+	A-	AA-	A	AA-	BBB+	A-	AA	A+	A+
20	A-	BBB+	BBB-	A+	AA-	BBB+	A-	AA	BBB+	AAA
21	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A
22	A-	A-	BBB	A+	AA-	BBB+	A-	AA	BBB+	AAA
23	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
24	AA+	A-	BBB-	A+	AAA	BBB+	A-	AA	BBB+	A+
25	AA+	A-	BBB-	AA	AA-	BBB+	A-	AA	BBB+	AAA
26	AA+	A-	BBB-	AAA	AA-	BBB+	A-	AA	BBB+	A+
27	A-	A-	BBB-	A+	AA-	BBB+	A-	AA-	BBB+	A+
28	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
29	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
30	BBB-	BBB+	BBB-	A+	AA-	BBB	A-	AA	BBB+	AAA
31	A-	A-	BBB	AAA	AA-	BBB+	A-	AA	BBB+	AA-
32	BBB+	BBB+	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
33	BBB	A-	BB	A+	A+	BBB+	A-	AA	BBB+	A+
34	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	BBB+	A+
35	A-	A-	BBB-	A+	AA-	BBB+	A-	AA	A-	AAA
Etc.	…	…	…	…	…	…	…	…	…	…

Annex 10: Values of bonds by rating and number of pieces

	DP	E.ON	Metro	VW	NIKE	Comm.	Bayer	Nestle	Danone	Oracle
	1000	1000	1000	10	500	1000	1000	500	5	500
1	1,093,035	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
2	1,108,538	1,173,249	976,853	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
3	1,099,127	1,173,249	927,243	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
4	1,101,903	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,115,133
5	1,105,132	1,173,249	986,737	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
6	1,080,764	1,173,249	970,324	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
7	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
8	1,099,127	1,173,249	970,857	1,094,097	1,068,835	895,744	1,060,761	1,185,854	1,130,682	1,117,341
9	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
10	1,108,538	1,173,249	986,737	1,090,693	1,069,454	895,744	1,060,761	1,185,854	1,133,675	1,117,341
11	1,108,538	1,173,249	979,776	1,093,040	1,069,454	901,600	1,060,444	1,185,854	1,130,682	1,117,341
12	1,093,035	1,173,249	970,857	1,093,040	1,068,835	895,744	1,060,761	1,185,854	1,130,682	1,117,341
13	1,108,538	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
14	1,105,132	1,173,249	970,857	1,090,693	1,068,835	895,744	1,060,761	1,185,854	1,130,682	1,117,341
15	1,108,538	1,172,632	970,857	1,093,040	1,069,454	901,600	1,060,761	1,185,854	1,130,682	1,117,341
16	1,101,903	1,173,249	970,857	1,093,040	1,069,769	895,744	1,060,761	1,185,854	1,130,682	1,118,210
17	1,101,903	1,171,774	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
18	1,105,132	1,173,249	970,857	1,093,040	1,068,835	901,600	1,060,761	1,185,854	1,130,682	1,117,341
19	1,108,538	1,173,249	986,737	1,090,693	1,069,454	895,744	1,060,761	1,185,854	1,135,862	1,117,341
20	1,105,132	1,171,774	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,118,210
21	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,115,133
22	1,105,132	1,173,249	976,853	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,118,210
23	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
24	1,108,538	1,173,249	970,857	1,093,040	1,070,076	895,744	1,060,761	1,185,854	1,130,682	1,117,341
25	1,108,538	1,173,249	970,857	1,093,852	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,118,210
26	1,108,538	1,173,249	970,857	1,094,097	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
27	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,693	1,130,682	1,117,341
28	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
29	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
30	1,093,035	1,171,774	970,857	1,093,040	1,069,454	893,290	1,060,761	1,185,854	1,130,682	1,118,210
31	1,105,132	1,173,249	976,853	1,094,097	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,815
32	1,101,903	1,171,774	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
33	1,099,127	1,173,249	958,860	1,093,040	1,068,835	895,744	1,060,761	1,185,854	1,130,682	1,117,341
34	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,130,682	1,117,341
35	1,105,132	1,173,249	970,857	1,093,040	1,069,454	895,744	1,060,761	1,185,854	1,133,675	1,118,210
Etc.	…	…	…	…	…	…	…	…	…	…

Annex 11: Probability distribution of the portfolio value

Scenario	Values	Frequency	Cumulative frequency	R1	R2
1	10,283,220	22	22	0.09%	0.09%
2	10,303,000	10	32	0.04%	0.13%
3	10,322,781	0	32	0.00%	0.13%
4	10,342,561	12	44	0.05%	0.18%
5	10,362,342	5	49	0.02%	0.20%
6	10,382,122	28	77	0.11%	0.31%
7	10,401,903	0	77	0.00%	0.31%
8	10,421,684	4	81	0.02%	0.32%
9	10,441,464	7	88	0.03%	0.35%
10	10,461,245	2	90	0.01%	0.36%
11	10,481,025	0	90	0.00%	0.36%
12	10,500,806	3	93	0.01%	0.37%
13	10,520,586	27	120	0.11%	0.48%
14	10,540,367	0	120	0.00%	0.48%
15	10,560,147	7	127	0.03%	0.51%
16	10,579,928	12	139	0.05%	0.56%
17	10,599,708	0	139	0.00%	0.56%
18	10,619,489	0	139	0.00%	0.56%
19	10,639,269	0	139	0.00%	0.56%
20	10,659,050	4	143	0.02%	0.57%
21	10,678,830	8	151	0.03%	0.60%
22	10,698,611	55	206	0.22%	0.82%
23	10,718,391	62	268	0.25%	1.07%
24	10,738,172	406	674	1.62%	2.70%
25	10,757,952	874	1548	3.50%	6.19%
26	10,777,733	194	1742	0.78%	6.97%
27	10,797,513	3918	5660	15.67%	22.64%
28	10,817,294	17191	22851	68.76%	91.40%
29	10,837,074	2149	25000	8.60%	100.00%
30	10,856,855	0	25000	0.00%	100.00%